Փուլային անցումները Քվանտային Հոլի երևույթում. Եզրային վիճակների դերը

Առաջատար հետազոտություններ. 21AG-1C024

Արա Սեդրակյան ,   Իլյա Գռուզբերգ (Օհայո ԱՄՆ)

Հրաչյա Բաբուջյան,   Շահանե Խաչատրյան,

Մխիթար Միրումյան,   Ելենա Ապրեսյան,

Արտաշես Մկրտչյան,   Արտավազդ Հարությունյան

Չենան Վեյի --փոստ. դոկ.

Հրանտ Թոփչյան ----> Ռուդիկ Բադալյան----> Չենան Վեյ

Տարվա ընթացքում տպագրվել է 5 աշխատանք

H. Topchyan , I. A. Gruzberg , W. Nuding, A. Klümper, and A. Sedrakyan-- PRB 110, L081112 (2024)

H. Topchyan , W. Nuding, A. Klümper, A. Sedrakyan- PRB 111, L100201 (2025)

H. Topchyan, V. Iugov, M. Mirumyan , T. Hakobyan, T.A. Sedrakyan, A. Sedrakyan-- SciPost Phys. 18, 068 (2025)

E. Apresyan, G. Sargsyan-*Regge symmetry of 6j-symbols of the Lorentz group*, Anal. Math. Phys. **15**, 113 (2025).

H. Babujian, M. Karowski, A. Sedrakyan- Factorization in deep inelastic scattering at Björken limit: Reduction to (1+1)D integrable models. ArXiv: 2503.11735

Գիտաժողովներ

Landau Week: Frontiers In Theoretical Physics-Yerevan 2025

Գալիլեո Գալիլեյ Կենտրոն, Ֆլորենցիա- Հրանտ Թոփչյան

Դուբայի Համալսարան--- Ելենա Ապրեսյան

Գործուղումներ

Վուպպերտալի համալսարան

Գալիլեո Գալիլեյ Կենտրոն, Ֆլորենցիա

Պեկինի Յաուի գերազանցության կենտրոն

Մետաղների Ինստիտուտ, Շենյանգ, Չինաստան

Z_3 and $Z_3 \otimes Z_3 \otimes Z_3$ symmetry protected topological paramagnets

1. Symmetry Protected Topological (SPT) phases: An extension of Landau's framework

2. Mathematical background

3. Gapless edge states as an hallmark of non-trivial SPT phases

4. Conformal properties of edge states. Open problems

1. Hrant Topchyan, Vasilii Iugov, Mkhitar Mirumyan, Shahane Khachatryan, Tigran Hakobyan, Tigran Sedrakyan -----JHEP 12. 129 (2023)

2. Hrant Topchyan, Vasilii Iugov, Mkhitar Mirumyan, Tigran Hakobyan, Tigran Sedrakyan, and Ara Sedrakyan -----arXiv: 2312.15095; SciPost----in press

The traditional Landau theory of phases and phase transitions classifies phases of matter based on spontaneous symmetry breaking (SSB) and the associated local order parameters. According to Landau, different phases correspond to different symmetries: when a system undergoes a phase transition, it typically breaks certain symmetries, and the nature of the broken symmetry characterizes the phase.

Can this approach be extended?   Yes, towards the topology.

Topological insulators are new states of quantum matter which can not be adiabatically connected to conventional insulators and semiconductors. They are characterized by a full insulating gap in the bulk and gapless edge or surface states which are protected by time-reversal symmetry. These topological materials have been theoretically predicted and experimentally observed in a variety of systems, including HgTe quantum wells, BiSb alloys, and Bi_2 Te_3 and Bi_2 Se_3 crystals.

The discovery of topological insulators and topological superconductors was a major breakthrough in the study of symmetry protected topological phases. Later, it was realized that such phenomena is not restricted to free fermion systems, but can be found in general interacting systems as well. A systematic study of interacting SPT phases followed, although the simplest interacting SPT model, the Haldane chain, has been known since the 1980s. It is Heisenberg model of spin 1.

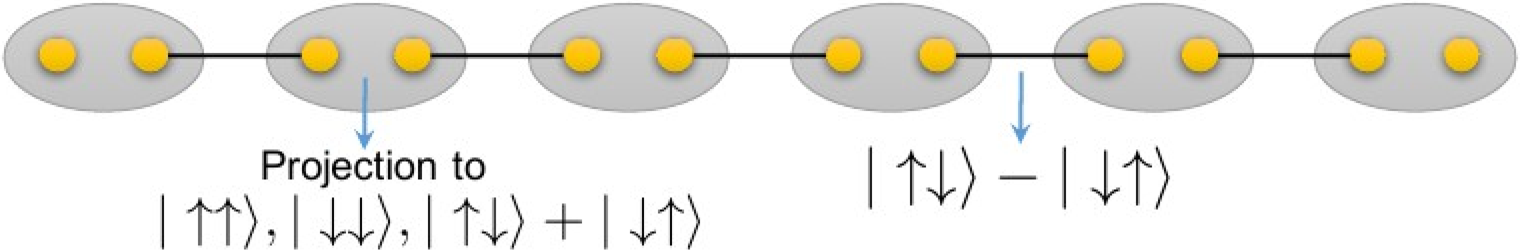
F. D. M. Haldane. Physics Letters A, 93:464, 1983.

It appeared, that spin one and all odd spin Heisenberg models are gapped, but they have zero energy edge states. How to see them? Example is

the simplest 1D SPT model, the AKLT model

I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki--Phys. Rev. Lett., 59(7):799–802 (1987)

Ground state is a product state and 4 fold degenerate



$$|\psi_{AKLT}\rangle = \prod_i P_1^i \prod_{i,i+1} \frac{1}{\sqrt{2}} \left(|\uparrow_r^i \downarrow_l^{i+1}\rangle - |\downarrow_r^i \uparrow_l^{i+1}\rangle \right)$$

where $P_1^i = |1\rangle\langle\uparrow_l^i\uparrow_r^i| + |-1\rangle\langle\downarrow_l^i\downarrow_r^i| + \frac{1}{\sqrt{2}}|0\rangle \left(\langle\uparrow_l^i\downarrow_r^i| + \langle\downarrow_l^i\uparrow_r^i| \right)$

Is the projection of two spin states back to spin 1

The system is gaped and we have massless states at the ends of chain

The definition of symmetry protected topological phases (SPT)

[Chen, Gu, Wen, 2010]

A local lattice Hamiltonian belongs to a symmetry protected topological (SPT) phase if obeys the following conditions:

- Short-range entangled, i. e. can be prepared from a product state by a finite-depth quantum circuit (FDQC).
- Symmetric: It is invariant under the onsite G symmetry.
- It has the unique gapped ground state.

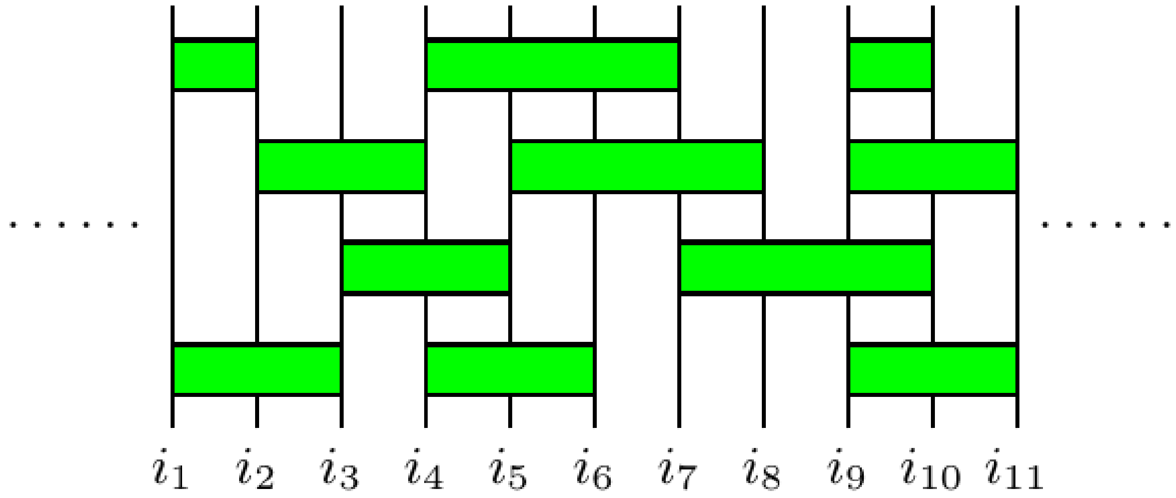
Different SPT phases

- Two SPT states belong to the same phase, if are connected by FDQC, composed of symmetric gates.
- An SPT state is trivial if it belongs to the same phase a product state,
- In contrast, a nontrivial SPT state cannot be disentangled by applying a FDQC with symmetric gates.

- They composed from local unitary (LU) transformations each given by a "Hamiltonian" $\tilde{H}(g) = \sum_i O_i(g)$,

$$U = \mathcal{T} e^{-i \int_0^1 dg \tilde{H}(g)}$$

- The FDQC example:



with local the unitaries

$$U_{123} = U^{i'_1 i'_2 i'_3}_{i_1 i_2 i_3}, \qquad U_{9,10} = U^{i'_9 i'_{10}}_{i_9 i_{10}}, \quad \ldots$$

In short terms

Consider spin Hamiltonian H(S), which is G-symmetric

$$[H(S), G] = 0$$

Consider unitary transformation of the Hamiltonian by G-symmetric U

$$H' = U H(S) U^{-1} = H(U S U^{-1}) = H(\tilde{S})$$

The spectrum of the system remains the same
but the ground state changes

$$\Psi_{GS}' = U \Psi_{GS}$$

If there is no all the way G-symmetric U(t), such that U(0) =1 while U(1) = U, then we have different SPT phases

Classification of SPT phases

- Symmetry protected topological orders in d dimension are classified by their symmetry group cohomology [Chen, Gu, Liu, Wen, 2011]

$$\mathcal{H}^{1+d}(G, U(1)), \qquad U(1) = \{e^{i\phi}\} \text{ is formed by GS wavefunction's phases}$$

What are the cohomologies of groups?

n- simplex $(g_1, \ldots, g_{n+1}) \sim (gg_1, \ldots, gg_{n+1}) \; . \in G \times G \times G \ldots = G^{n+1}$

$$G \leftleftarrows G \times G \;\substack{\longleftarrow\\\longleftarrow\\\longleftarrow}\; G \times G \times G \;\substack{\longleftarrow\\\longleftarrow\\\longleftarrow\\\longleftarrow}\; \cdots$$

Semisimplicial complex

Boundary operation

$$\delta(g_1, g_2, \ldots. g_n) = \sum (-1)^i (g_1, \ldots \check{g}_i, \ldots g_n)$$

group of formal linear combinations of n-simplexes with U(1) coefficients called chains $C_n(G;U(1))$

Consider now linear g-invariant functions $\varphi : \underbrace{G\times ....\times G}_{n+1} \to U(1)$ on chains

$$\varphi(g\, g_0, g\, g_2, ... g\, g_n) = g\, \varphi(g_0, ... g_n)$$

$Functions\ v = e^{i\varphi}, called\ n-dimensional\ cochains, form\ a\ group\ of\ cochains\ C^n(G;U(1))$

Boundary operation

$$\delta\varphi(g_0,\ldots,g_{k+1}) = \sum_{l=0}^{k+1}(-1)^l \varphi(g_0,\ldots,\hat{g}_l,\ldots,g_{k+1}),$$

- The δ operator is nilpotent: $\delta^2\varphi = 0\ mod\ 2\pi, \to \delta^2 v = 1$

For convenience one also can define $\phi - cochain$

$$\phi(g_1, g_1, \ldots. g_n) = g_1 \varphi(1, g_1, g_1 g_2, g_1 g_2 g_3, \ldots. g_1 g_2 \cdot g_n)$$

Then the boundary operator δ for it become

$$\delta \phi(g_1, g_1, \ldots. g_n) = g_1 \phi(g_2, g_3, \ldots g_n) + \sum_1^n (-1)^j \phi(g_1, \ldots g_j^{-1} g_{j+1} \ldots g_n)$$

$$+ (-1)^n \phi(g_1, \ldots. g_n)$$

Again $\delta^2 \phi = 0,$ $\omega = e^{i\phi}$

Operator δ defines cohomology groups

For the group elements in multiplicative exponential form $\omega = e^{i\phi}$ $\phi \equiv n$

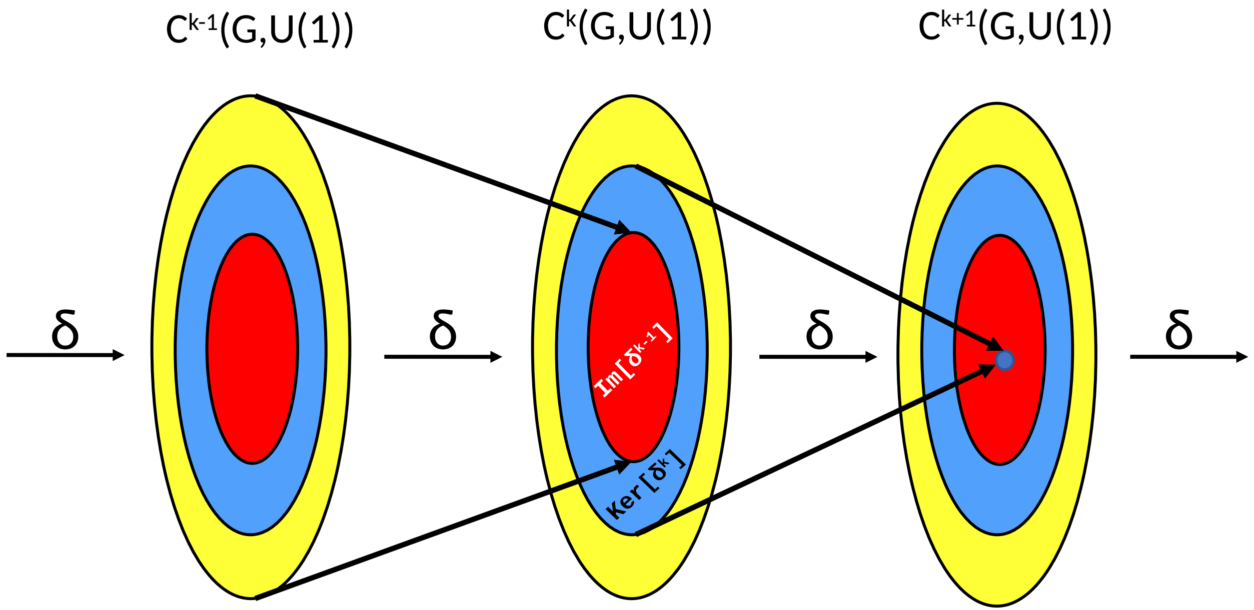
$$\delta\omega_1(n_1,n_2) = \frac{\omega_1(n_2)\omega_1(n_1)}{\omega_1(n_1+n_2)},$$

$$\delta\omega_2(n_1,n_2,n_3) = \frac{\omega_2(n_2,n_3)\omega_2(n_1,n_2+n_3)}{\omega_2(n_1+n_2,n_3)\omega_2(n_1,n_2)},$$

$$\delta\omega_3(n_1,n_2,n_3,n_4) = \frac{\omega_3(n_2,n_3,n_4)\omega_3(n_1,n_2+n_3,n_4)\omega_3(n_1,n_2,n_3)}{\omega_3(n_1+n_2,n_3,n_4)\omega_3(n_1,n_2,n_3+n_4)}.$$

Pentagon identity $\longrightarrow$ $\delta\omega_3(n_1,n_2,n_3,n_4) = 1$

ω_3 is an exact form



Group of cohomology

$$H^k(G,U(1)) = \frac{\mathrm{Ker}\,\delta^k}{\mathrm{Im}\,\delta^{k-1}}$$

SPT phases are classified by the cohomology group $H^{d+1}(G, U(1))$

Chen, Gu, Liu, Wen, 2011

In a chain models d=1 and we have $H^2(G, U(1))$

In a plain models d=2 and we have $H^3(G, U(1))$

Simple SPT model in d=2 with Z_2 symmetry was constructed in

M. Levin and Z.-C. Gu. Braiding statistics approach to symmetry-protected topological phases. Phys. Rev. B, 86:115109 (2012)

Ising model on triangular lattice in paramagnetic phase

$$H_I = -\sum_p X_p \qquad X_p = \sigma_p^x$$ Pauli matrix

- There are two different SPT phases with $G = \mathbb{Z}_2$ symmetry in $d = 2$ dimension, since

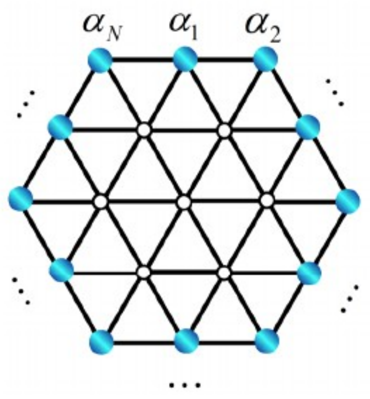
$$\mathcal{H}^{1+d}(\mathbb{Z}_2, U_1) = \mathbb{Z}_2$$

- The trivial phase is given by paramagnetic Ising model

$$H_0 = -\sum_p X_p,$$

with

$$|\mathrm{gs}\rangle = |00\ldots 0\rangle = \sum_{\alpha_i=\uparrow,\downarrow} |\alpha_1 \ldots \alpha_N\rangle,$$



where $|0\rangle, |1\rangle = |\uparrow\rangle \pm |\downarrow\rangle$ relate X and Z eigenstates.

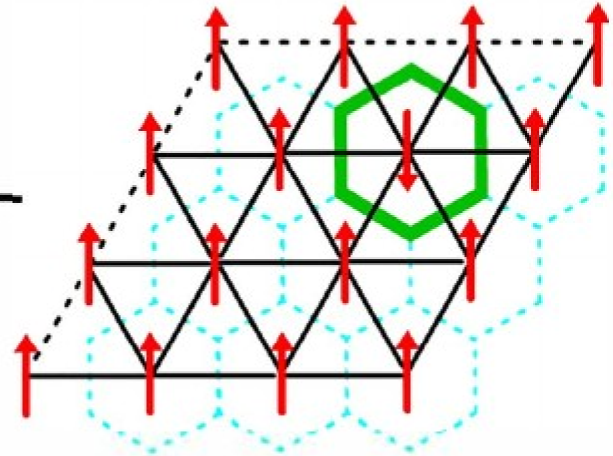
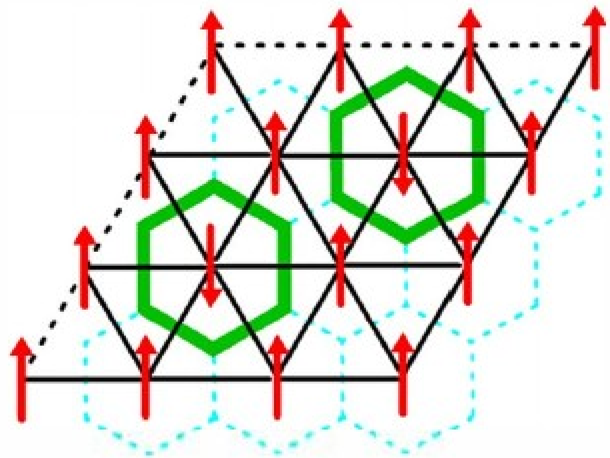
- Invariant under $G = \mathbb{Z}_2$ group with $X = \prod_p X_p$.

- There is unitary map from H_0 to H_1 by

$$U = (-1)^{\sum_{\text{triangles}} n_p n_q n_r + \sum_{\text{links}} n_p n_q + \sum_{\text{vertexes}} n_p} = \prod_{p,q,r} CCZ_{pqr} \prod_{p,q} CZ_{pq} \prod_{p} Z_p$$

- Domain wall description of the ground state,

$$|\text{gs}\rangle = \sum_{\alpha_p = \uparrow,\downarrow} (-1)^{\#(\text{domain walls})} |\alpha_1 \dots \alpha_N\rangle = |\uparrow \dots \uparrow\rangle - |\downarrow\uparrow \dots \uparrow\rangle + \dots$$

$= -$  $+$  $+ \dots$

Three-state Potts model

1. Hrant Topchyan, Vasilii Iugov, Mkhitar Mirumyan, Shahane Khachatryan, Tigran Hakobyan, Tigran Sedrakyan -----JHEP 12. 129 (2023)

2. Hrant Topchyan, Vasilii Iugov, Mkhitar Mirumyan, Tigran Hakobyan, Tigran Sedrakyan, and Ara Sedrakyan -----arXiv: 2312.15095; SciPost Phys. 18, 068 (2025)

$$H_P = \gamma \sum_{\mathbf{r}\in 2D,\mu} \left(\varepsilon^{S^z_{\mathbf{r}}}\varepsilon^{-S^z_{\mathbf{r}+\mu^{\mathbf{a}}}} + \varepsilon^{-S^z_{\mathbf{r}}}\varepsilon^{S^z_{\mathbf{r}+\mu}}\right) + \sum_{\mathbf{r}\in 2D}\left(X^+_{\mathbf{r}} + X^-_{\mathbf{r}}\right),$$

$$X^+_{\mathbf{r}} = (X^-_{\mathbf{r}})^\dagger = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad S^z_{\mathbf{r}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad \varepsilon = e^{\frac{2\pi i}{3}}$$

Algebra

$$X^\pm_{\mathbf{r}}\varepsilon^{S^z_{\mathbf{r}}} = \varepsilon^{\mp 1}\varepsilon^{S^z_{\mathbf{r}}}X^\pm_{\mathbf{r}}, \qquad (X^+_{\mathbf{r}})^3 = (X^-_{\mathbf{r}})^3 = 1.$$

Colored three-state Potts model in paramagnetic phase

G = Z3 × Z3 × Z3

$$H_0 = -\sum_{p,\alpha} \left(X_p^{\alpha} + X_p^{+\alpha} \right) \qquad \alpha = 1,2,3$$

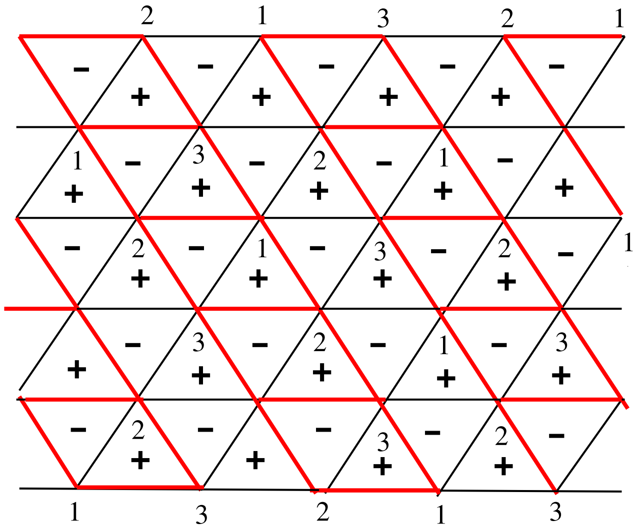
Z_3 Ising symmetry group is $$X^{\pm} = \prod_{\mathbf{m},i=1,2,3} X^{\pm}_{\mathbf{m},i},$$

SPT phases will be defined by the cohomology group

$$H^3(Z_3 \times Z_3 \times Z_3, U(1)) = (. \times Z_3)^7$$

SPT states via cohomologies

We should form Z3 symmetric unitary operator U as a product of operators defined on triangles



a)

$$U = \prod_{\langle pqr \rangle} U\left(n_p^i, n_q^j, n_r^k\right)$$

$$U\left(n_p^i, n_q^j, n_r^k\right) = \left(.\times Z_3\right)^7$$

I, j, k = 1,2,3 are color indexes

$$n_p \equiv S_p^z = -1, 0, +1$$

represents Z_3 in additive form

We have 7 choices of colors i, j, k on triangles: {i,i,i} ---3 ; {i,i,j}---3 ; {I,j,k}--- 1

Topologically non-trivial co-cycles of $H^3(Z_3 \times Z_3 \times Z_3, U(1)) = (. \times Z_3)^7$ are

$$\varepsilon = e^{\frac{2\pi i}{3}}$$

$$\omega_3^{123}(n_1, n_2, n_3) = \varepsilon^{n_1^{(1)} n_2^{(2)} n_3^{(3)}}.$$

$$\omega_3^{i}(n_1, n_2, n_3) = \varepsilon^{n_1^{(i)} n_2^{(i)} n_3^{(i)} \left(n_2^{(i)} + n_3^{(i)}\right)}, \qquad i = 1, 2, 3$$

$$\omega_3^{ij}(n_1, n_2, n_3) = \varepsilon^{n_1^{(i)} n_2^{(j)} n_3^{(j)} \left(n_2^{(j)} + n_3^{(j)}\right)}$$ i,j={1,2,2}, {1, 3,3}, {2,3,3}

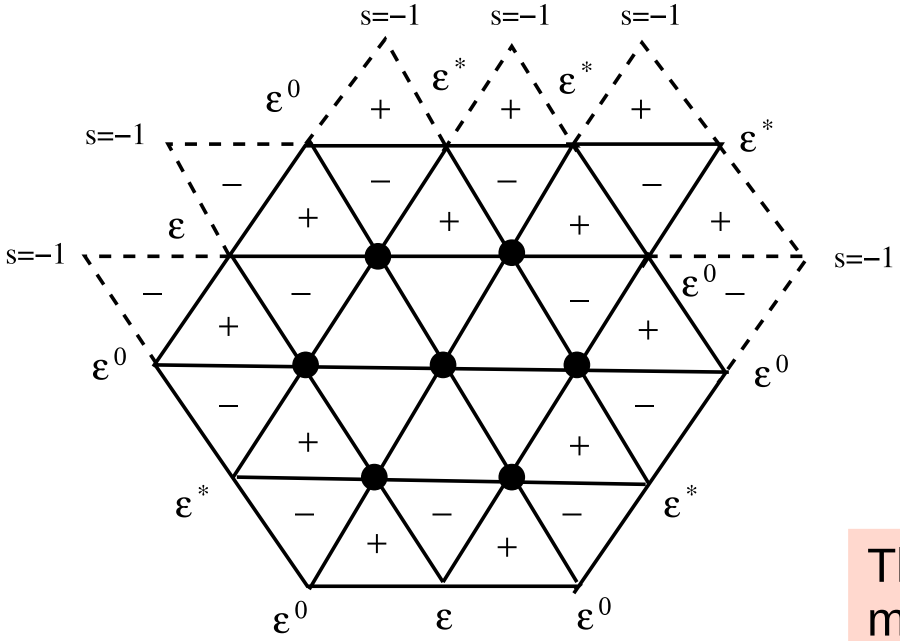
We will consider now the diagonal case 1,2,3 when all colors are different.
It is diagonal embedding of Z_3 into (x Z_3)^7

$$U(n_1, n_2, n_3) = v_3(0, n_1^{\Delta}, n_3^{\Delta}, n_3^{\Delta},) = \omega_3(n_1^{\Delta}, n_2^{\Delta} - n_1^{\Delta}, n_3^{\Delta} - n_2^{\Delta})$$

$$U = \prod_{\Delta} \nu_3(0, n_1^{\Delta}, n_2^{\Delta}, n_3^{\Delta})^{S(\Delta)} = \prod_{\Delta} \omega_3(n_1^{\Delta}, n_2^{\Delta} - n_1^{\Delta}, n_3^{\Delta} - n_2^{\Delta})^{S(\Delta)}$$

$$U = \prod_{\Delta} \varepsilon^{S(\Delta) n_1 n_2 n_3 (n_2 - n_1)}$$

Here Δ represents the triangle. $S(\Delta) = \pm 1$ is a sign factor associated triangles in a staggered way



(b)

The staggering of sign factor was included to ensure the symmetry of U under Z_3 Ising

$$X^{\pm} = \prod_{\mathbf{m}, i=1,2,3} X^{\pm}_{\mathbf{m},i},$$

$$U\, X^{\pm \cdot} = X^{\pm \cdot}\, U$$

This U creates non-trivial SPT phase, which will manifest itself in formation of massless edge state on the boundary of the sample.

Finally, all terms together give following symmetric boundary Hamiltonian

$$H^{(a)}_{edge} = -\sum_p \left(X^+_p + \Omega_a^{-1} X^+_p \Omega_a + \Omega_a X^+_p \Omega_a^{-1} + H.c. \right)$$

$$H^{(a)}_{edge} = -\sum_p \Big(X^+_p + Z^+_{p-1} X^+_p Z^+_{p+1} + Z^-_{p-1} X^+_p Z^-_{p+1} + H.c. \Big),$$

This Hamiltonian is massless. To see it consider

$$H^{(a)}_{edge}(\lambda_1, \lambda_2, \lambda_3) = -\sum_p \left(\lambda_1 X^+_p + \lambda_2 Z^+_{p-1} X^+_p Z^+_{p+1} + \lambda_3 Z^-_{p-1} X^+_p Z^-_{p+1} + H.c. \right).$$

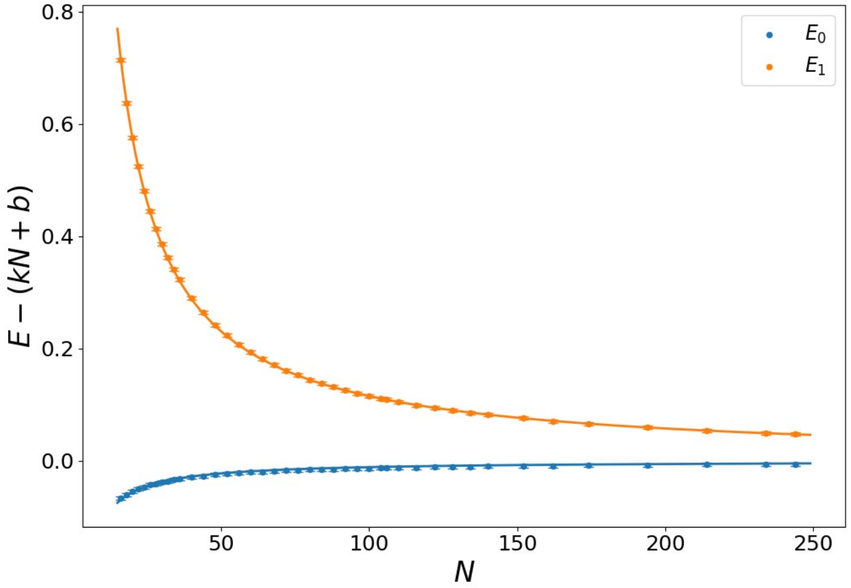
This Hamiltonian is self trial under

$$X^{d\pm}_p = \Omega_a^{-1} X^\pm_p \Omega_a = Z^\pm_{p-1} X^\pm_p Z^\pm_{p+1}, \quad Z^{d\pm}_p = Z^\pm_p$$

Therefore, at equal couplings it should be massless

Conformal properties of low lying excitations of the model. What CFT is responsible for them?

We have calculated gap of the model numerically

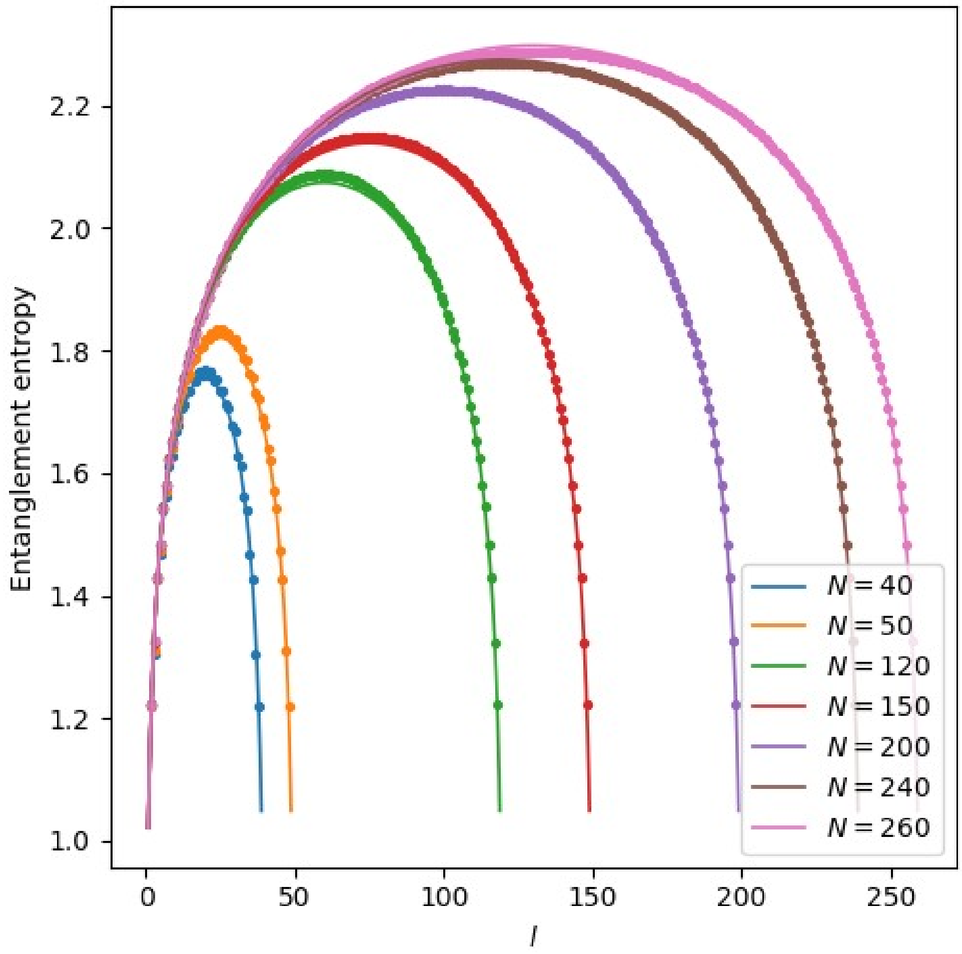


$$\Delta_N = \frac{2\pi\, 0.66625}{N} + \mathcal{O}(10^{-2}).$$

According to J.L.Cardy-Nucl.Phys. B270, 186 (1986)

X= 0.66625= 2/3 is the conformal dimension of the scaling operator concerned with correlation length

Entanglement entropy of the model



According to
J.L.Cardy-Nucl.Phys. B270, 186 (1986)

Finite size behavior of the entanglement entropy of the model defines its central charge

$$S_N(l) = a + \frac{c}{6}\log\left[\frac{1}{\pi}\sin\left[\pi\frac{l}{N}\right]\right]$$

l= 1……N

Numerical calculations gave

C = 1.73

Summary and open problems

1. We have constructed SPT model with $Z_3 \times Z_3 \times Z_3$ and Z_3 symmetries

2. Numerically have argued that they are massless and at critical point they have CFT limit as coset $\frac{SU(3)_k}{SU(2)_k}$ model with k=2 and k=1 respectively

3. What are the statistical properties of the excitations in the edge model? How they are linked with primary fields in CFT? What kind of defects of the standard model leads to corresponding fields/excitations?