Արտաքին դաշտերով և տոպոլոգիայով մակածված քվանտային երևույթներ դաշտի տեսությունում և կոնդենսացված միջավայրերի ֆիզիկայում

Ա. Ա. Սահարյան

Ֆիզիկայի ինստիտուտ, Երևանի պետական համալսարան

Ամենամյա ամփոփիչ գիտաժողով – 25, 5-11 հոկտեմբերի, 2025թ., Դիլիջան

Առաջատար հետազոտությունների աջակցության գիտական թեմա 21AG-1C047

Թեմա՝ «Արտաքին դաշտերով և տոպոլոգիայով մակածված քվանտային երևույթներ դաշտի տեսությունում և կոնդենսացված միջավայրերի ֆիզիկայում»

1. Սահարյան Արամ
2. Ավագյան Ռոլանդ
3. Հարությունյան Գոհար
4. Հակոբյան Տիգրան
5. Օհանյան Վադիմ
6. Պետրոսյան Տիգրան
7. Սիմոնյան Դավիթ
8. Վարոսյան Ռաֆի
9. Ամիրաղյան Լուսիկ /ընդգրկված սեպտեմբերից 2025թ./

Հետազոտական ուղղությունները

- Արտաքին գրավիտացիոն դաշտերով, տարածության կորությամբ, սահմաններով ու տոպոլոգիայով պայմանավորված երևույթներ դաշտի քվանտային տեսությունում կոնդենսացված միջավայրերի ֆիզիկայի 2D մոդելներում

- Ճշգրիտ ինտեգրվող ցանցային մոդելներ մագնիսական համակարգերում կիրառություններով

- Նյութի տոպոլոգիական փուլեր, կիրառությունները քվանտային տեխնոլոգիաներում

- Լիցքավորված մասնիկների ճառագայթումը անհամասեռ միջավայրերում (ՀՀ ԳԱԱ Ֆիզիկայի կիրառական պրոբլեմների հետ համագործակցությամբ)

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Գիտաժողովների մասնակցություն

1. Evolving Universe: Theory and Observations. Starobinsky Memorial Conference, 7-12 October, 2024, Yerevan, Armenia.
2. International Conference on Algebra, Logic, and Their Applications, 13-19 October, 2024, Yerevan, Armenia.
3. The 12th International Symposium ”Optics and its Applications” (OPTICS-12), 15-19 October, 2024, Yerevan, Armenia.
4. 18th Russian Gravitational Conference (RUSGRAV18) 25-29, 2024, November, Kazan, Russia.
5. DPG Spring Meeting of the Condensed Matter Section, 16 – 21 March, 2025, Regensburg, Germany
6. XVII Armenian-Georgian Joint Astronomical Colloquium, 5-9 May, 2025, Byurakan, Armenia.
7. Quantum Theory and Symmetries (QTS-13), 28 July - 1 August, 2025, Yerevan, Armenia.
8. Modern Physics of Compact Stars and Relativistic Gravity, 23-26 September, 2025, Yerevan, Armenia.

Properties of the Fulling-Rindler vacuum

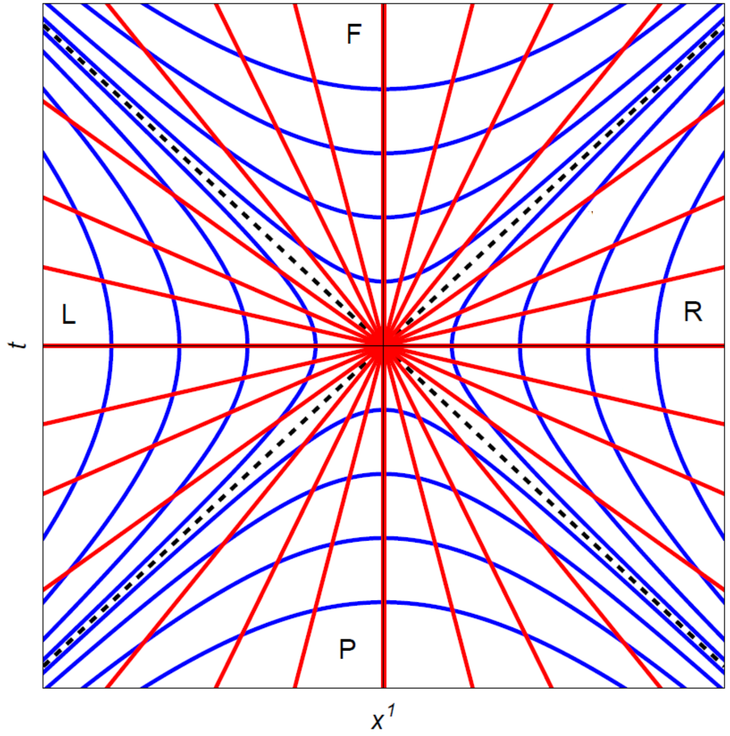
Importance of QFT in Rindler spacetime

- Observer dependence of the vacuum and particle notions is among the important lessons from quantum field theory on curved spacetimes
- Classical example of two inequivalent vacuum states in the Minkowski spacetime are the Minkowski and Fulling-Rindler vacua ⇨ Vacuum states for inertial and uniformly accelerating observers, respectively
- Interest to QFT in Rindler spacetime is motivated by several reasons:
  - Principal questions of quantization of fields in geometries having horizons
  - Rindler geometry is simple enough to find exact solutions in different problems of quantum field theory
  - Rindler metric approximates the black hole geometry in the near horizon limit and the roots of a number of quantum field theoretical phenomena around black holes can be found in the Rindler physics (example is the relation between the Unruh effect and Hawking radiation)
  - Being a background with horizons, the Rindler geometry is an interesting arena to investigate the phenomena of quantum entanglement

Problems considered

- Polarization of the fermionic Fulling-Rindler (FR) vacuum

Rindler line element $ds_{\rm R}^2 = g_{\mu\nu}dx^\mu dx^\nu = \rho^2 d\tau^2 - d\rho^2 - d\mathbf{x}^2,\ \mathbf{x} = (x^2, x^3, \ldots, x^D),$



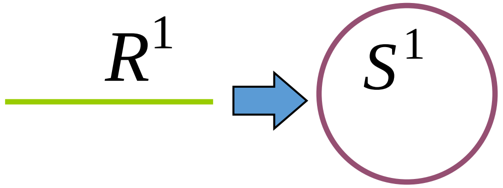
- Tranfsormation to Minkowskian coordinates

$t = \rho \sinh \tau,\ x^1 = \rho \cosh \tau, \quad ds_{\rm M}^2 = dt^2 - d\mathbf{z}^2,\ \mathbf{z} = (x^1, \mathbf{x})$

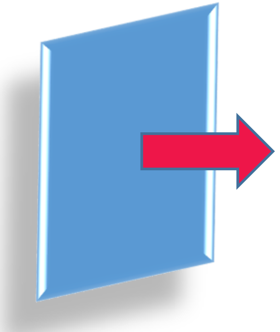
- Dirac field $(i\gamma^\mu \nabla_\mu - m)\psi = 0,$

- Quantum effects of nontrivial topology in Rindler spacetime with compact dimensions (scalar and Dirac fields)

A part of dimensions $\mathbf{x} = (x^2, x^3, \ldots, x^D),$ is compactified to a torus



- The fermionic Casimir effect for a mirror uniformly accelerated through the FR vacuum



Planar boundary moving with constant acceleration

Polarization of the fermionic FR vacuum: Fermion condensate

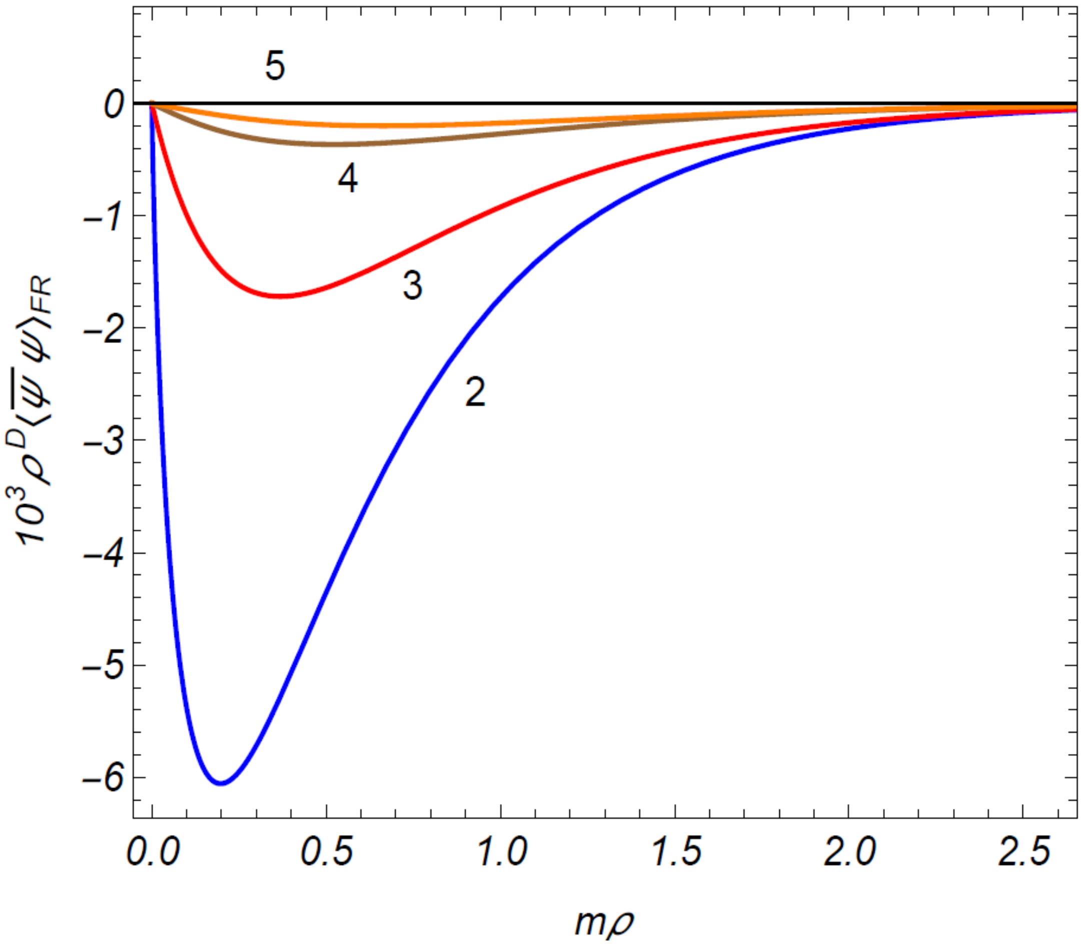
- Renormalized fermionic condensate

Spatial dimension

$$\langle \bar{\psi}\psi \rangle_{\mathrm{FR}} = -\frac{2Nm^D}{(2\pi)^{\frac{D+3}{2}}} \int_0^\infty du \, \frac{u \sinh u}{u^2 + \pi^2/4} f_{\frac{D-1}{2}} (2m\rho \cosh u) \qquad f_\nu(z) = z^{-\nu} K_\nu(z)$$

- Fermionic condensate is negative for a massive field, vanishes for a massless field

$1/\rho$ ⇨ Proper acceleration of an observer



Polarization of the fermionic FR vacuum: Energy-momentum

- Vacuum expectation value of the energy-momentum tensor $\langle T_{\mu\nu} \rangle$
- Vacuum energy density

$$\langle T_0^0 \rangle_{\rm FR} = \frac{2Nm^{D+1}}{(2\pi)^{\frac{D+3}{2}}} \int_0^\infty du \, \frac{u^2 - \pi^2/4}{(u^2+\pi^2/4)^2} f_{\frac{D+1}{2}} (2m\rho \cosh u) \cosh u$$

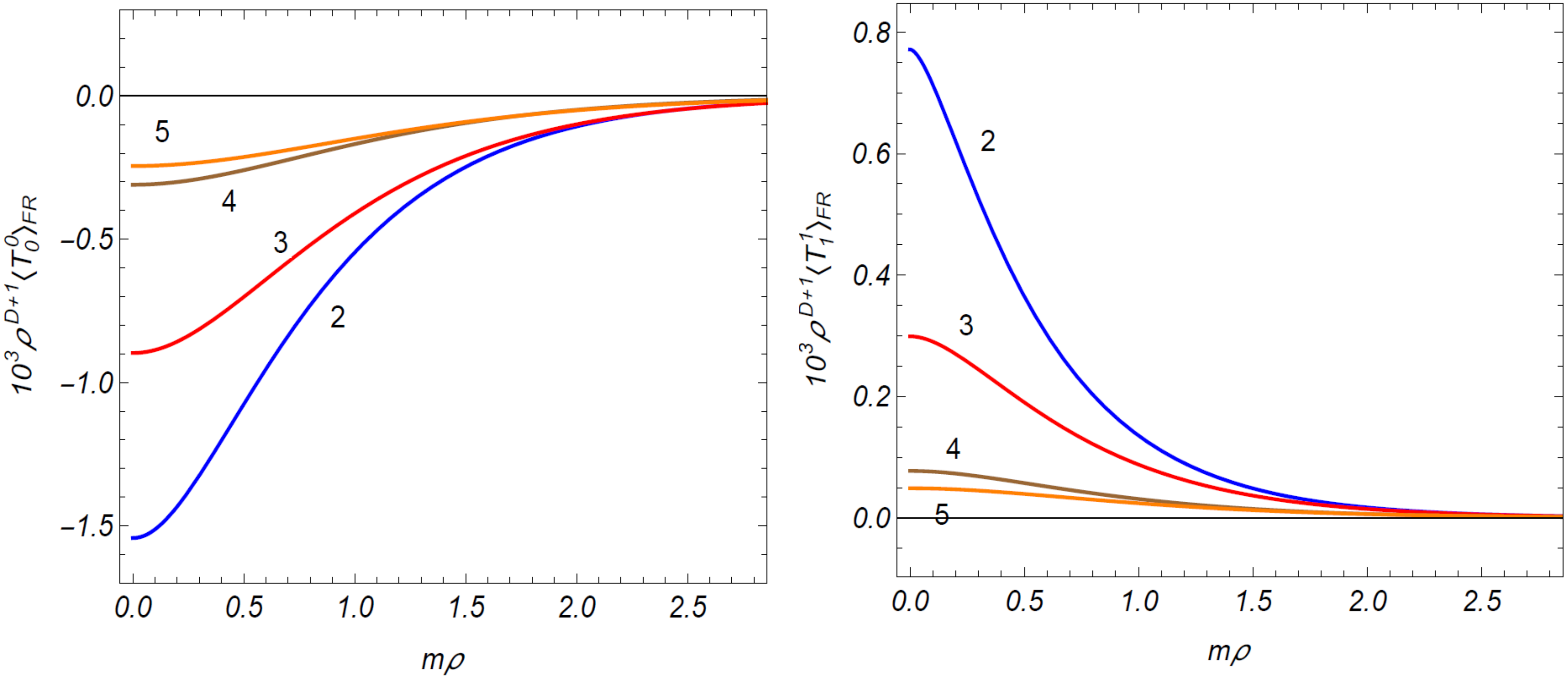
Vacuum energy density is negative

- Vacuum stresses (= - vacuum pressure)

$$\langle T_l^l \rangle_{\rm FR} = \frac{2Nm^{D+1}}{(2\pi)^{\frac{D+3}{2}}} \int_0^\infty du \, \frac{u \sinh u}{u^2+\pi^2/4} f_{\frac{D+1}{2}} (2m\rho \cosh u)$$

Vacuum pressures are isotropic (not the case for a scalar field)

Vacuum energy density and pressures



Vacuum energy-momentum tensor for a massless field

- For a massless field

$$\langle T_\mu^\nu \rangle_{\rm FR} = \frac{A_D^{(\rm ferm)}}{\rho^{D+1}} {\rm diag}\left(-1, \frac{1}{D}, \ldots, \frac{1}{D}\right), \quad A_D^{(\rm ferm)} = \frac{2^{-D} N}{\pi^{\frac{D}{2}} \Gamma\left(\frac{D}{2}\right)} \int_0^\infty d\omega \, \frac{\omega^D B_D(\omega)}{e^{2\pi\omega} - (-1)^D}$$

$$B_n(\omega) = \prod_{l=1}^{l_n} \left[1 + \left(\frac{l - \{n/2\}}{\omega}\right)^2\right], \quad l_n = n/2 - 1 + \{n/2\}$$

Radiation type equation of state *Pressure=Energy density/D* with negative energy density

- Special case *D=3* $\langle T_\mu^\nu \rangle_{\rm FR} = \frac{1}{\pi^2 \rho^4} \int_0^\infty d\omega \, \frac{\omega\left(\omega^2 + \frac{1}{4}\right)}{e^{2\pi\omega} + 1} {\rm diag}\left(-1, \frac{1}{3}, \ldots, \frac{1}{3}\right)$

In terms of the proper energy measured by an observer with acceleration $1/\rho$

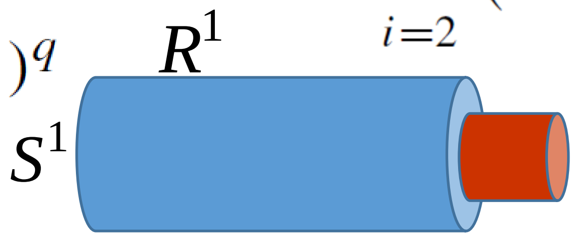
$\left(e^{2\pi\rho\varepsilon_\rho} + 1\right)^{-1} \Leftarrow$ thermal nature with the Unruh temperature $T = 1/(2\pi\rho)$

- In even number of spatial dimensions the thermal factor for Dirac field is of bosonic type $\left(e^{2\pi\rho\varepsilon_\rho} - 1\right)^{-1}$

Quantum effects of nontrivial topology

- Rindler spacetime with compact dimensions $ds^2 = \xi^2 d\tau^2 - d\xi^2 - \sum_{i=2}^{D} \left(dx^i\right)^2,$

$$x^\mu = \left(x^0 = \tau, x^1 = \xi, \mathbf{x}_p, \mathbf{x}_q\right)$$ ⇦topology⇨ $R^p \times (S^1)^q$



- Quantum scalar field $\left(g^{\mu\nu} D_\mu D_\nu + m^2\right)\varphi = 0,\ D_\mu = \nabla_\mu + ieA_\mu,$ ⇦ External gauge field

  We take $A_\mu = \text{const}$ (interpreted in terms of the magnetic flux)

- Quasiperiodicity conditions along compact dimensions

$$\varphi(t, \xi, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\alpha_l} \varphi(t, \xi, \mathbf{x}_p, \mathbf{x}_q)$$

  Momentum along l-th compact dimension $k_l = (2\pi n_l + \tilde{\alpha}_l)/L_l$

- Physical characteristics of the FR vacuum state

  - ❖Vacuum expectation value (VEV) of the field squared
  - ❖VEV of the energy-momentum tensor
  - ❖Vacuum current density

Vacuum current density

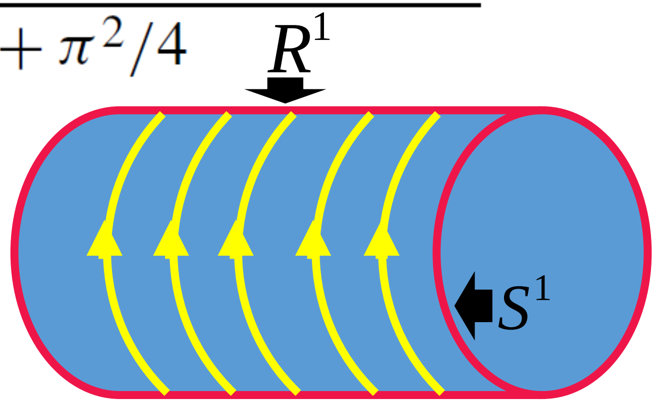
- Results will be described for the current density in the model with a single compact dimension
- Current density along the compact dimension (length *L*)

$$\langle j^D\rangle = \langle j^D\rangle_{\mathrm{M}} - \frac{4em^{D+1}L}{(2\pi)^{(D+1)/2}}\sum_{n=1}^{\infty} n\sin(n\tilde{\alpha}_D)\int_0^{\infty}dx\,\frac{f_{(D+1)/2}(m\sqrt{4\xi^2\cosh^2x+n^2L^2})}{x^2+\pi^2/4}$$

Current in Minkowski vacuum

$$\tilde{\alpha}_l = \alpha_l + eA_lL_l$$

$$\langle j^D\rangle_{\mathrm{M}} = \frac{4em^{D+1}L}{(2\pi)^{(D+1)/2}}\sum_{n=1}^{\infty} n\sin(n\tilde{\alpha}_D)f_{\frac{D+1}{2}}(nmL)$$



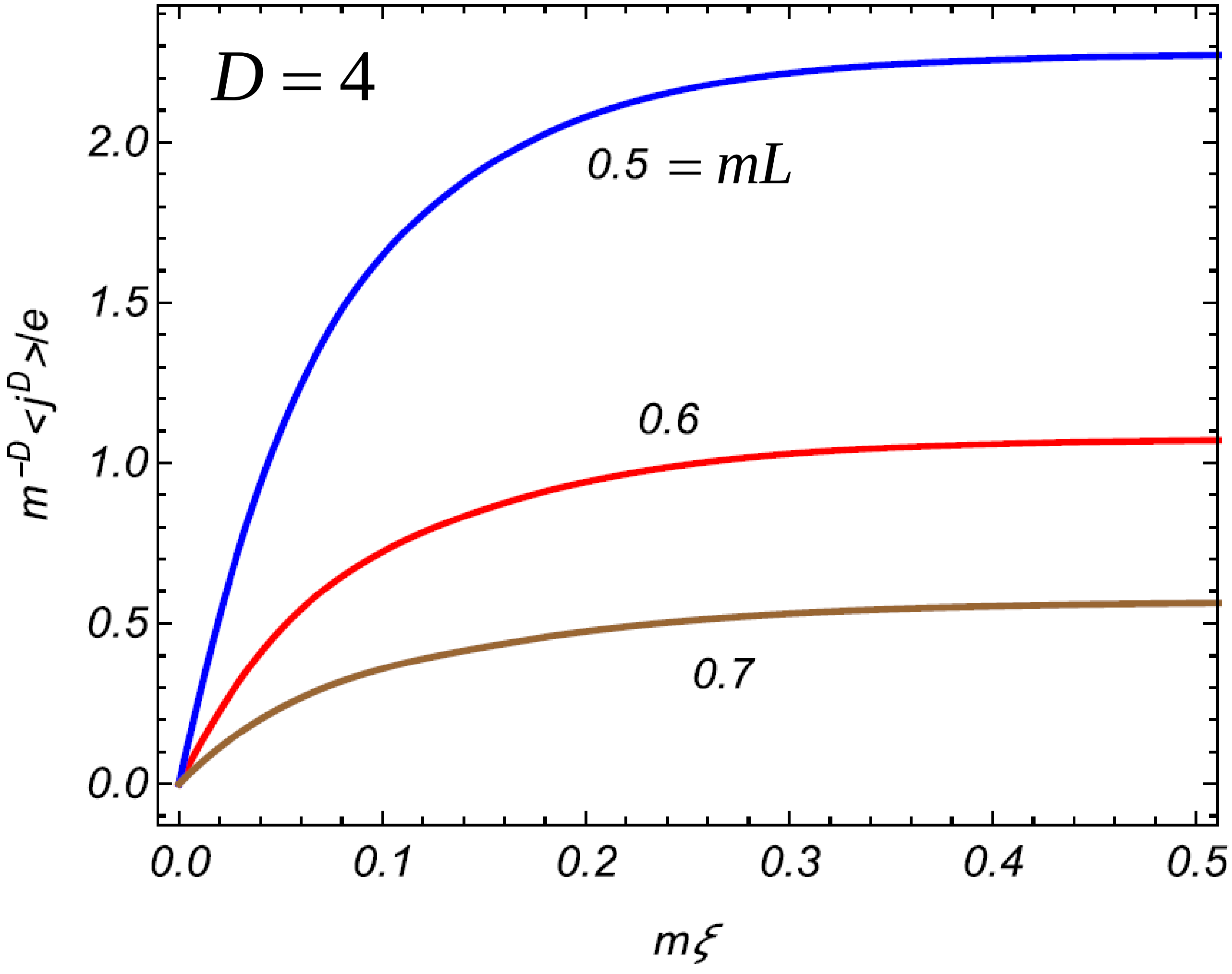
- Massless field

$$\langle j^D\rangle = \langle j^D\rangle_{\mathrm{M}} - 2eL\frac{\Gamma((D+1)/2)}{\pi^{(D+1)/2}}\sum_{n=1}^{\infty} n\sin(n\tilde{\alpha}_D)\int_0^{\infty}dx\,\frac{(4\xi^2\cosh^2x+n^2L^2)^{-\frac{D+1}{2}}}{x^2+\pi^2/4}$$

Current in Minkowski vacuum

$$\langle j^D\rangle_{\mathrm{M}} = \frac{2e\Gamma((D+1)/2)}{\pi^{(D+1)/2}L^D}\sum_{n=1}^{\infty}\frac{\sin(n\tilde{\alpha}_D)}{n^D}$$

Vacuum current density



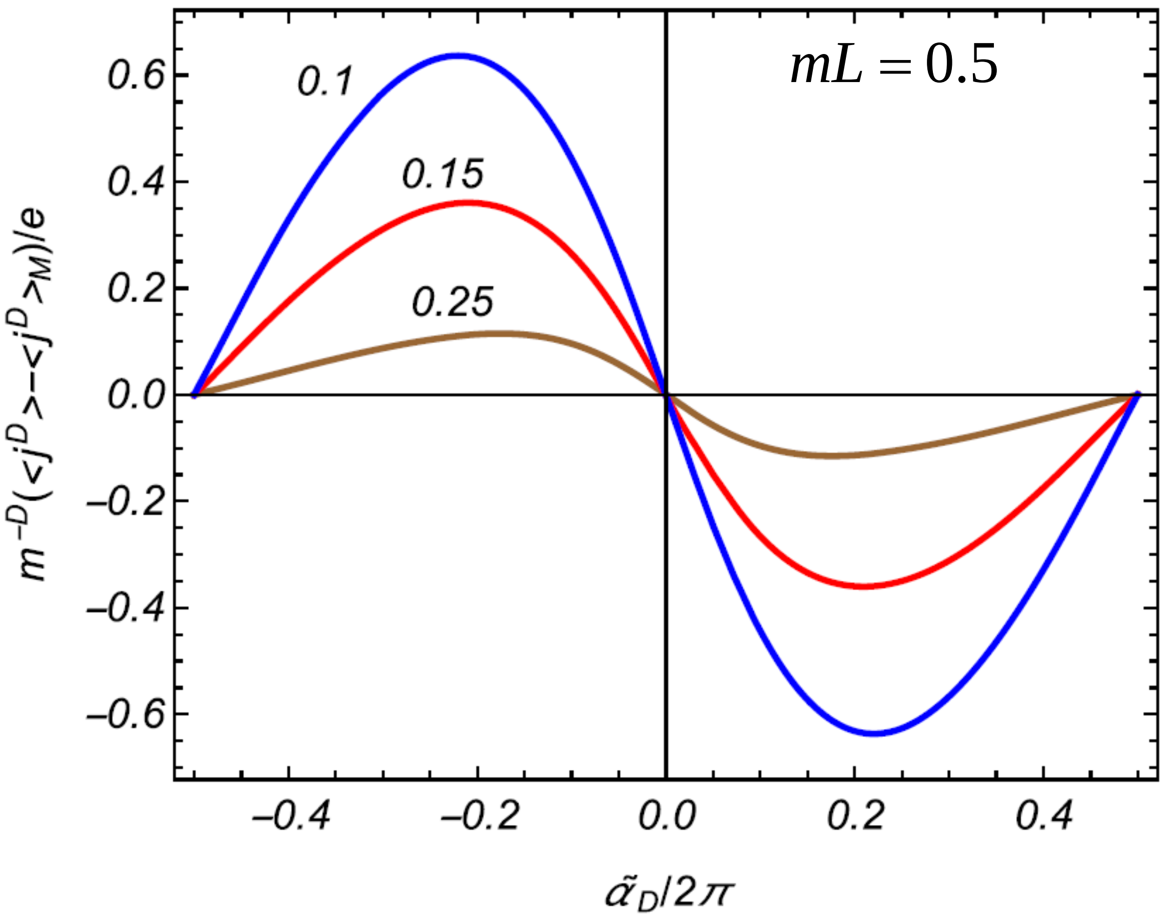
$\tilde{\alpha}_D / 2\pi = 0.2$

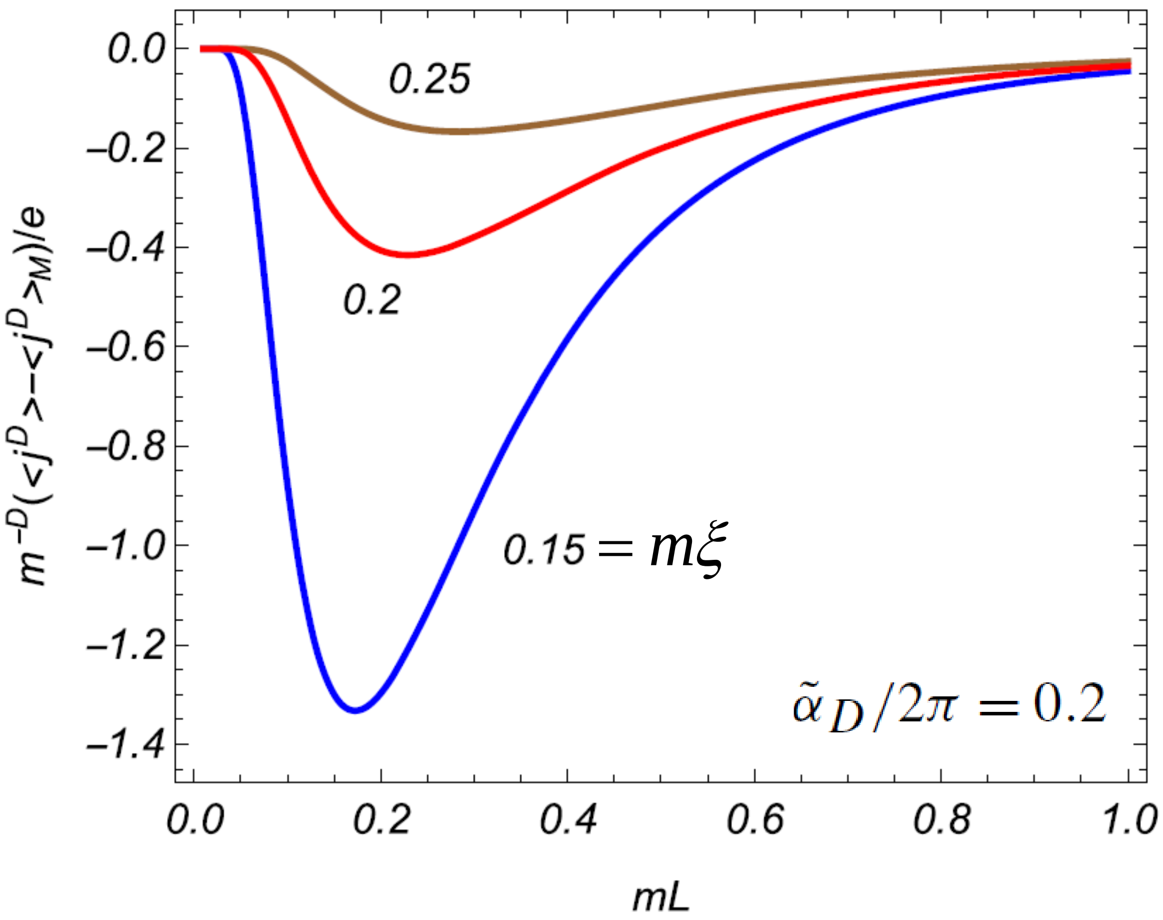
The current density along compact dimensions is a periodic function of the magnetic flux enclosed by those dimensions

Vanishes on the Rindler horizon

Obtained results describe the vacuum currents near the horizon of cylindrical black holes

Vacuum current density



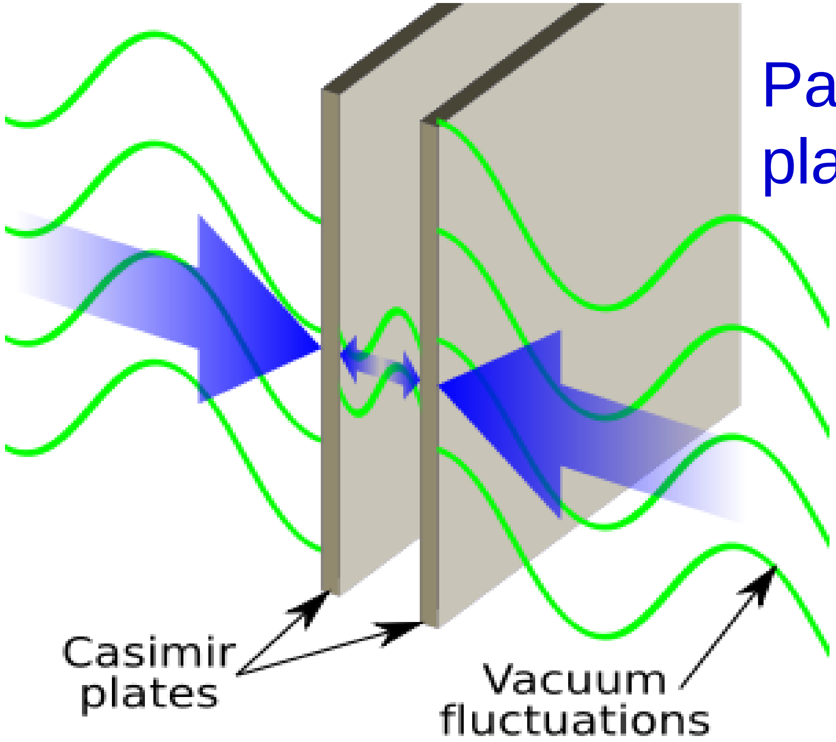


Fermionic Casimir effect in the FR vacuum

Casimir effect:

Imposition of boundary conditions on the field operator leads to the change of the spectrum for vacuum (zero-point) fluctuations

As a result the vacuum expectation values of physical observables are changed → Casimir effect



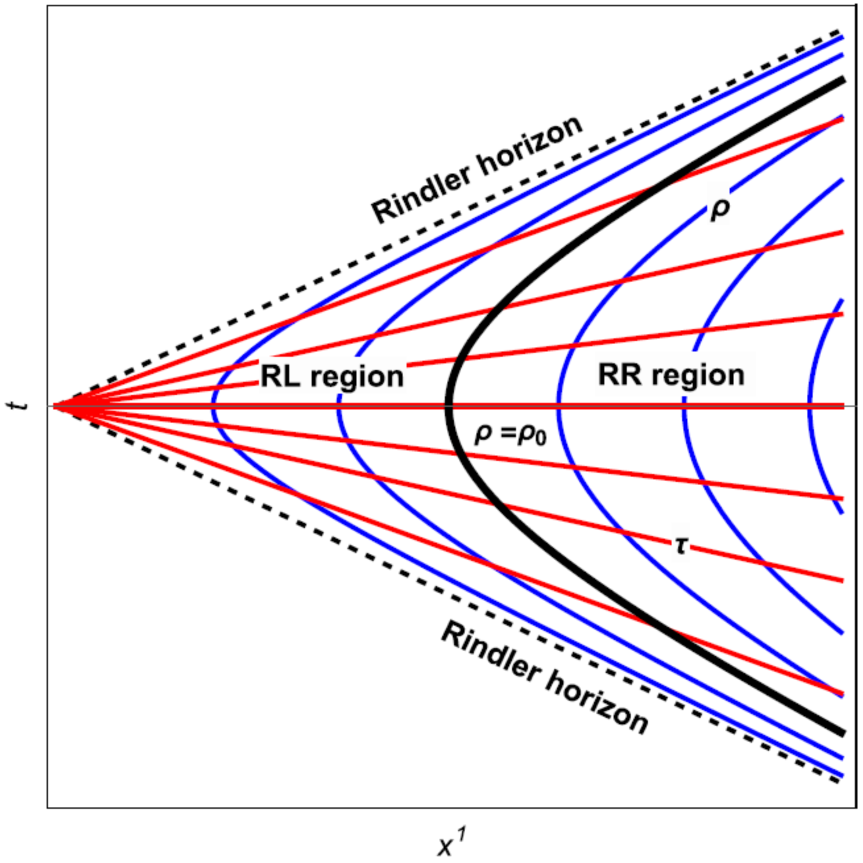
Parallel metallic plates

Casimir force $$\frac{F_C}{S} = -\frac{\partial}{\partial a}\frac{E_C}{S} = -\frac{\hbar c \pi^2}{240 a^4}$$

H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948)

Fermionic Casimir effect in the FR vacuum

- Dynamical Casimir effect: Boundaries moving with acceleration in the Minkowski vacuum create radiation
- We consider the influence of a planar boundary moving with constant acceleration on the properties of the FR vacuum for the Dirac field (spin 1/2)



- Dirac field $(i\gamma^{\mu}\nabla_{\mu} - m)\psi = 0, \ \nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu},$
- Bag boundary condition $(1 + in_{\mu}\gamma^{\mu})\psi = 0, \ \rho = \rho_0$
- Physical characteristics of the FR vacuum
  - Fermion condensate
  - VEV of the energy-momentum tensor

$$\langle\bar{\psi}\psi\rangle_{\rm ren} = \langle\bar{\psi}\psi\rangle_0^{\rm ren} + \langle\bar{\psi}\psi\rangle_{\rm b}$$

$$\langle T_{\mu}^{\mu}\rangle_{\rm ren} = \langle T_{\mu}^{\mu}\rangle_0^{\rm ren} + \langle T_{\mu}^{\mu}\rangle_{\rm b}$$

Boundary-induced contributions

Fermion condensate in RR-region

$$\langle\bar{\psi}\psi\rangle_{\mathrm{b}} = \frac{2^{1-D}mN}{\pi^{\frac{D+1}{2}}\Gamma\left(\frac{D-1}{2}\right)}\int_m^{\infty} d\lambda\,\lambda\left(\lambda^2-m^2\right)^{\frac{D-3}{2}}\int_{1/2}^{\infty} d\nu\,\frac{\bar{I}_{\nu}(\lambda\rho_0)}{\bar{K}_{\nu}(\lambda\rho_0)}\left[K_{\nu}^2(\lambda\rho)-K_{\nu-1}^2(\lambda\rho)\right]$$

Notations: $\bar{K}_{\nu}(x)=K_{\nu}(x)+K_{\nu-1}(x),\ \ \bar{I}_{\nu}(x)=I_{\nu}(x)-I_{\nu-1}(x)$ Modified Bessel functions

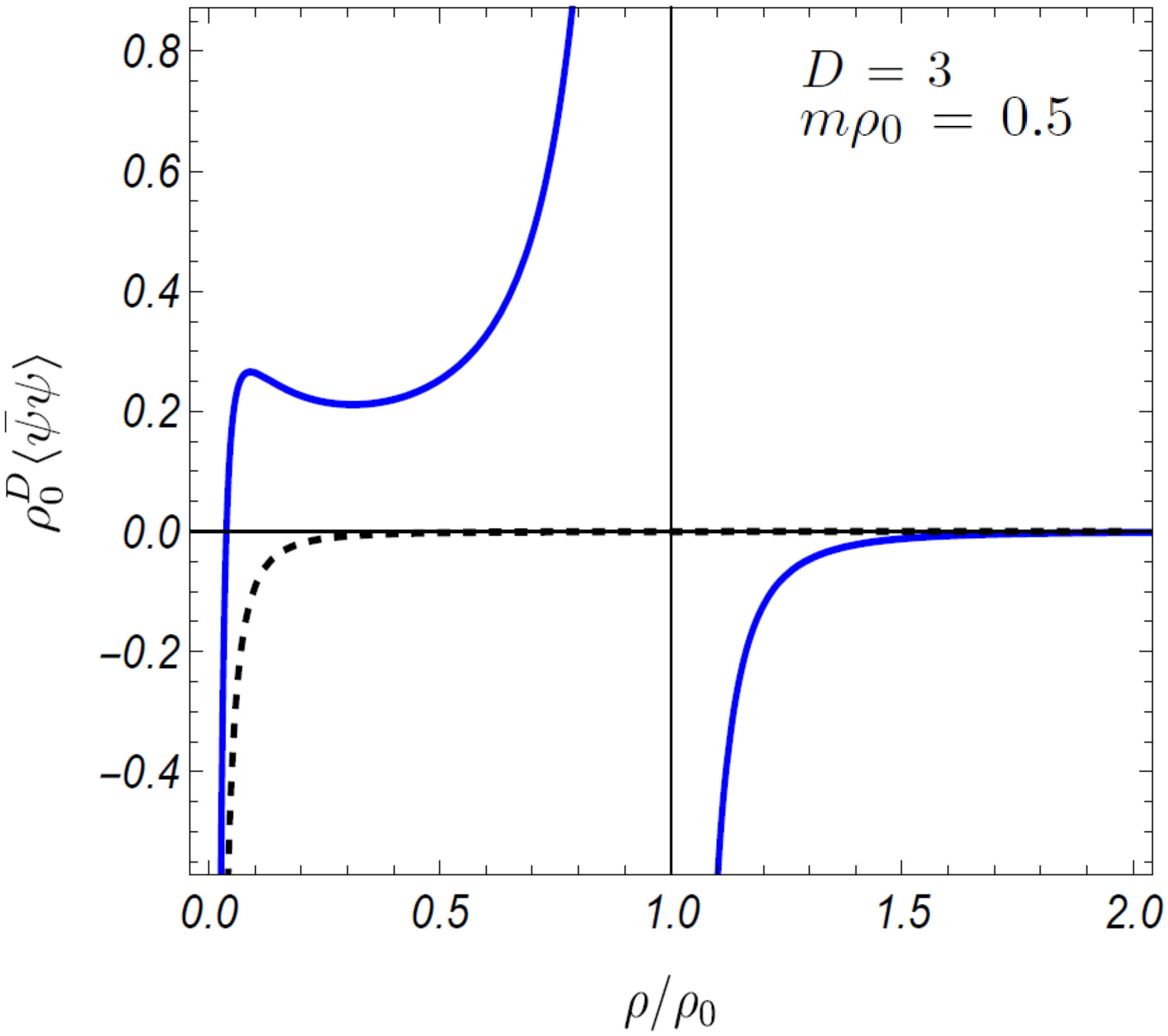
Energy-momentum tensor in RR-region

$$\langle T_{\mu}^{\mu}\rangle_{\mathrm{b}} = \frac{2^{1-D}N}{\pi^{\frac{D+1}{2}}\Gamma\left(\frac{D-1}{2}\right)}\int_m^{\infty} d\lambda\,\lambda\left(\lambda^2-m^2\right)^{\frac{D-3}{2}}\int_{1/2}^{\infty} d\nu\,\frac{\bar{I}_{\nu}(\lambda\rho_0)}{\bar{K}_{\nu}(\lambda\rho_0)}F_{\mu}[K_{\nu}(\lambda\rho)]$$

$$\begin{aligned}
F_0\left[G_{\nu}(\lambda\rho)\right] &= \delta_G\frac{2\lambda}{\rho}\left(\nu-\frac{1}{2}\right)G_{\nu}(\lambda\rho)\,G_{\nu-1}(\lambda\rho)\,,\\
F_1\left[G_{\nu}(\lambda\rho)\right] &= \delta_G\lambda^2\left[G_{\nu}'(\lambda\rho)\,G_{\nu-1}(\lambda\rho)-G_{\nu}(\lambda\rho)\,G_{\nu-1}'(\lambda\rho)\right]\\
F_l\left[G_{\nu}(\lambda\rho)\right] &= \frac{\lambda^2-m^2}{D-1}\left[G_{\nu-1}^2(\lambda\rho)-G_{\nu}^2(\lambda\rho)\right],
\end{aligned}$$

Fermion condensate and energy-momentum tensor in RL-region is obtained by the replacements $I_{\nu}(x)\rightleftarrows K_{\nu}(x)$

Boundary-induced contributions



Շնորհակալություն