Topology and quantum chaos in strongly correlated systems

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Topology and quantum chaos in strongly correlated systems

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- Disordered systems over flatband featuring lattices
- Disorder-induced SYK systems on Moire lattices (MATBG)
- Sytems with symmetry-protected topological phases
- Scale-free disordered networks

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Topology and quantum chaos in strongly correlated systems

- Topchyan, H., Iugov, V., Mirumyan, M., Hakobyan, T., Sedrakyan, T. A., & Sedrakyan, A. G. (2025). *Two-dimensional topological paramagnets protected by Z_3 symmetry: Properties of the boundary Hamiltonian*. SciPost Phys., 18, 068

- Topchyan, H., Nuding, W., Klümper, A., & Sedrakyan, A. (2025). *Harris-Luck criterion in the plateau transition of the integer quantum Hall effect*. Phys. Rev. B, 111(10), L100201

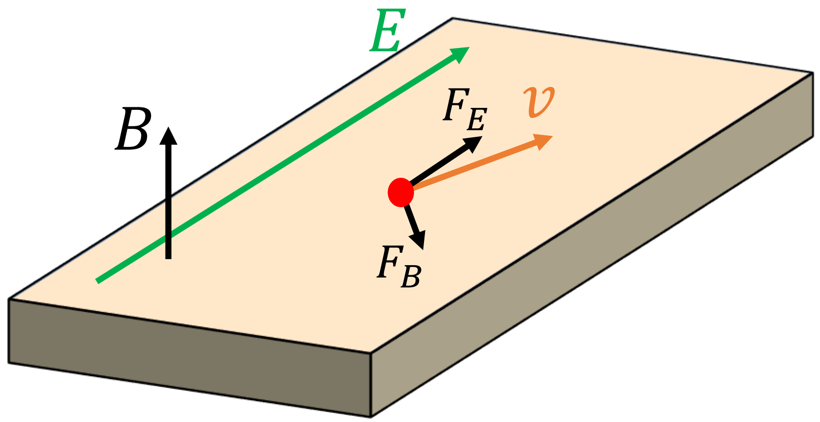
- Jafari-Zadeh, B., Wei, C., & Sedrakyan, T. A. (2025). *Chiral vortex-line liquid of three-dimensional interacting Bose systems with moat dispersion*. Phys. Rev. B, 111(24), 245130

- Tarafdar, A., & Sedrakyan, T. A. (2025). *Quantization and quantum oscillations of the sublattice charge order in Dirac insulators*. arXiv.2506.06681

Harris-Luck criterion in the Integer quantum Hall effect

- Quasiclassical picture of IQHE and Chalker-Coddington model
- Random Network (RN) models: induction of gravity
- Harris Criterion and scale-free networks

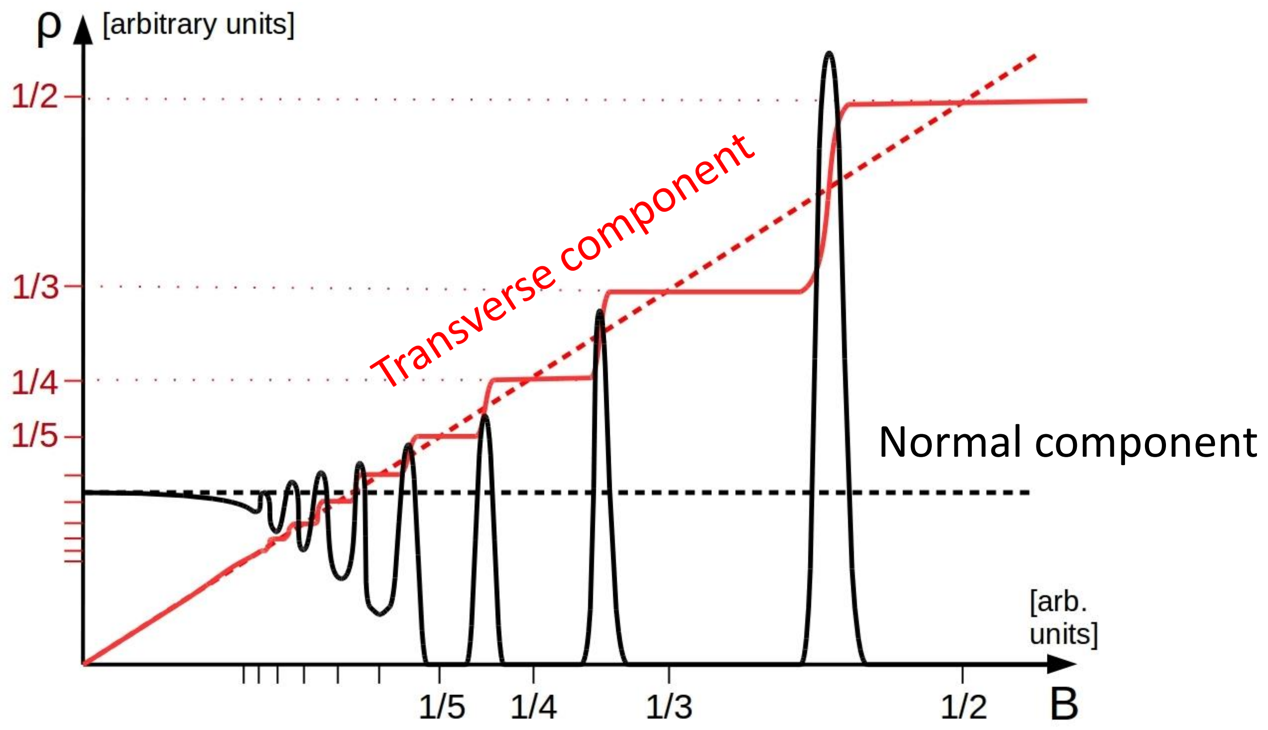
Classical theory and the strong field experiment



$$\vec{j} = \sigma\left(\vec{E} + \frac{\vec{j}\times\vec{B}}{\rho}\right) \qquad \vec{j} = \hat{\sigma}\vec{E}$$

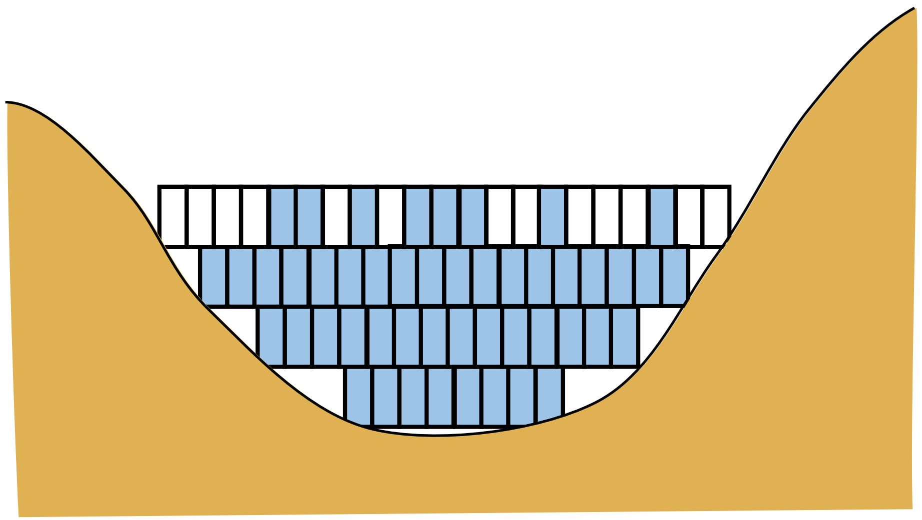
$$\hat{\rho} = \hat{\sigma}^{-1} = \sigma^{-1}\begin{pmatrix} 1 & k \\ -k & 1 \end{pmatrix} \qquad k = \frac{\sigma B}{\rho}$$

Normal Transverse



The mechanisms behind the effect

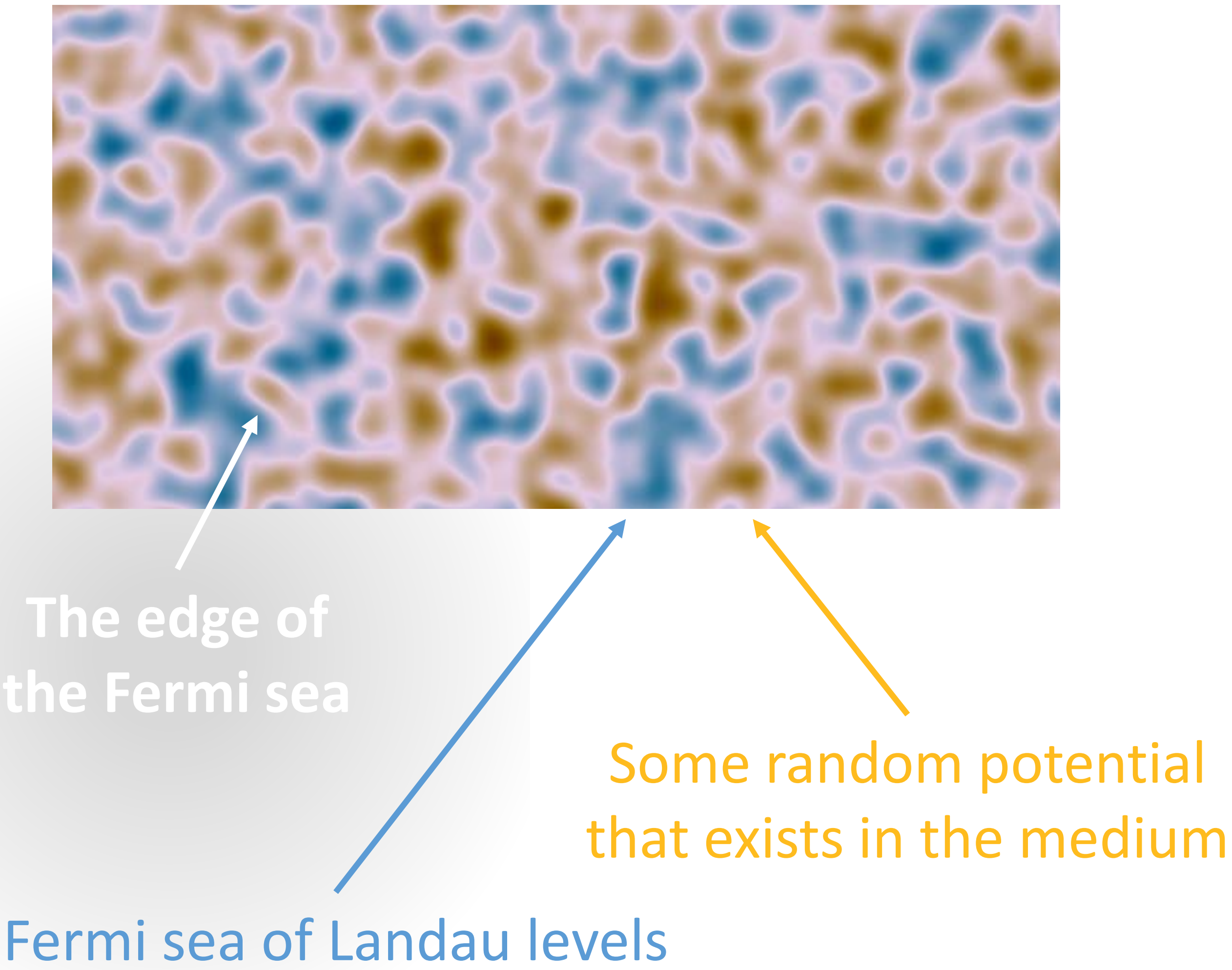
1. Condensation to Landau levels



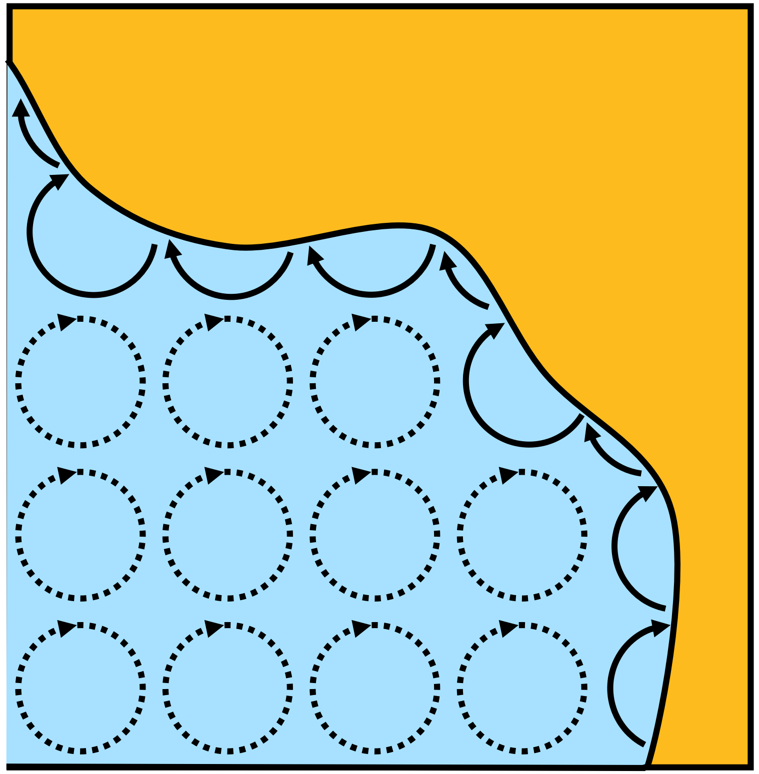
State “widths” decrease with increasing field

Conductance will occure when the edge states are partially occupied

2. Randomness of the medium



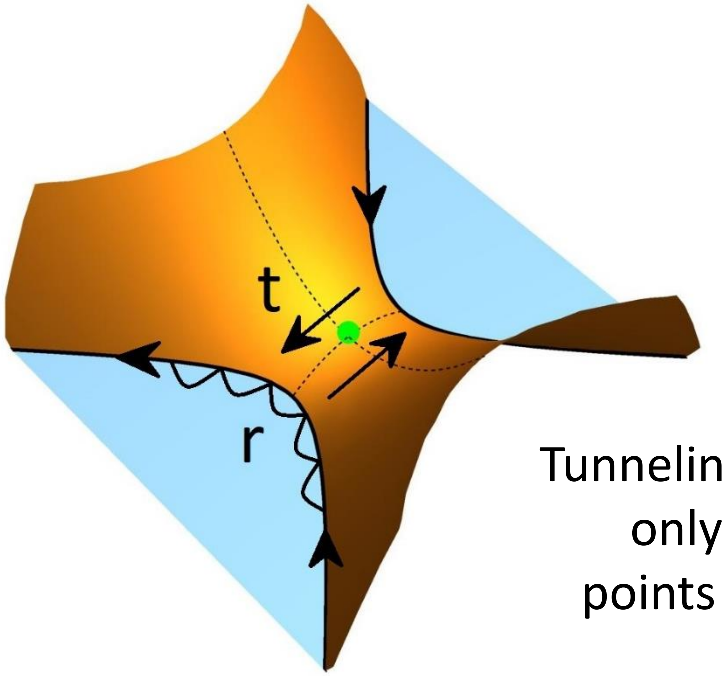
The edge states and tunneling conductivity



Semi-classical image of electrons moving pattern in the Fermi sea

The edge states are still localized

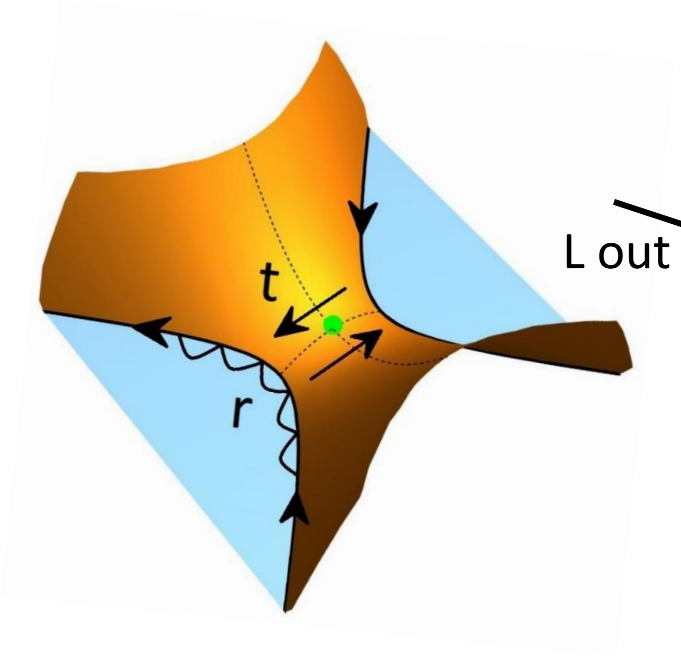
Fermi sea = many Fermi lakes

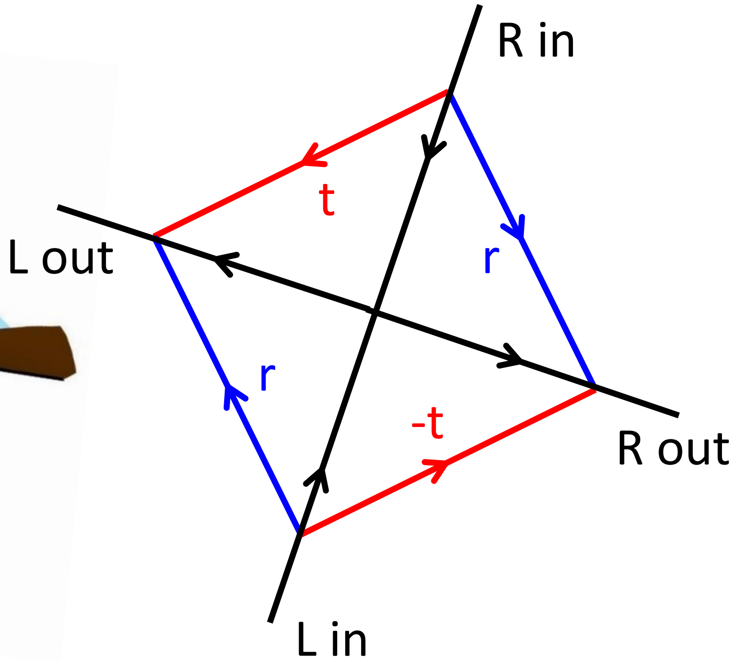


Tunneling happens almost only at the saddle points of the potential

The edge state electrons delocalize due to tunneling and thus create conductivity

Chalker-Coddington model





Scattering matrix

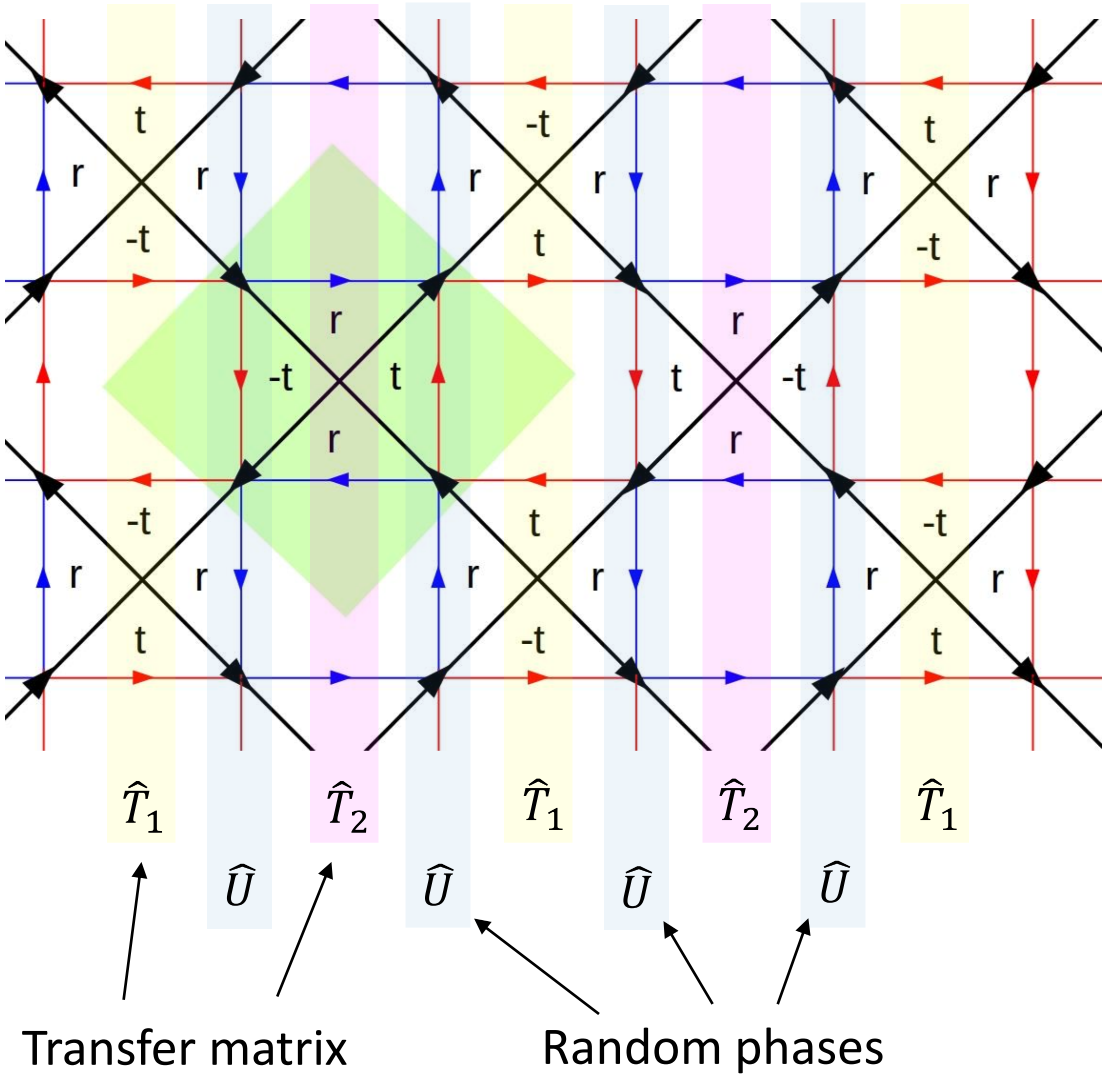
$$S : \begin{pmatrix} L \\ R \end{pmatrix}_{in} \rightarrow \begin{pmatrix} L \\ R \end{pmatrix}_{out}$$

$$S = \begin{pmatrix} r & t \\ -t & r \end{pmatrix}$$

Transfer matrix

$$T : \begin{pmatrix} out \\ in \end{pmatrix}_{L} \rightarrow \begin{pmatrix} in \\ out \end{pmatrix}_{R}$$

$$T = \begin{pmatrix} 1/t & r/t \\ r/t & 1/t \end{pmatrix}$$



Random phases create Anderson localization and thus allow critical behavior

Comparison to experiment

$\langle\psi_0\psi_N\rangle \sim e^{-N/N_0}$ ⟵ Correlation length

$N_0 = (x - x_c)^{-\nu}$ ⟵ Localization length index

Experimental value: $\nu = 2.38 \pm 0.06$ (W.Li et.al. *PRL*, *102*, 216801 (2009))

Calculations have finite size

Particle energy x System width M

Lyapunov exponents $\Gamma(M, x)$

Finite size scaling

ν

The index value for CC model:

$\nu = 2.593 \pm 0.006$: Ohtsuki, et.al. *PRB 80*, 041304 (2009)

$\nu = 2.62 \pm 0.01$: M. Amado, et.al. *PRL 107*, 066402 (2011)

$\nu = 2.57 \pm 0.02$: J.P.Dahlhaus, et.al. *PRB 84*, 115133 (2011)

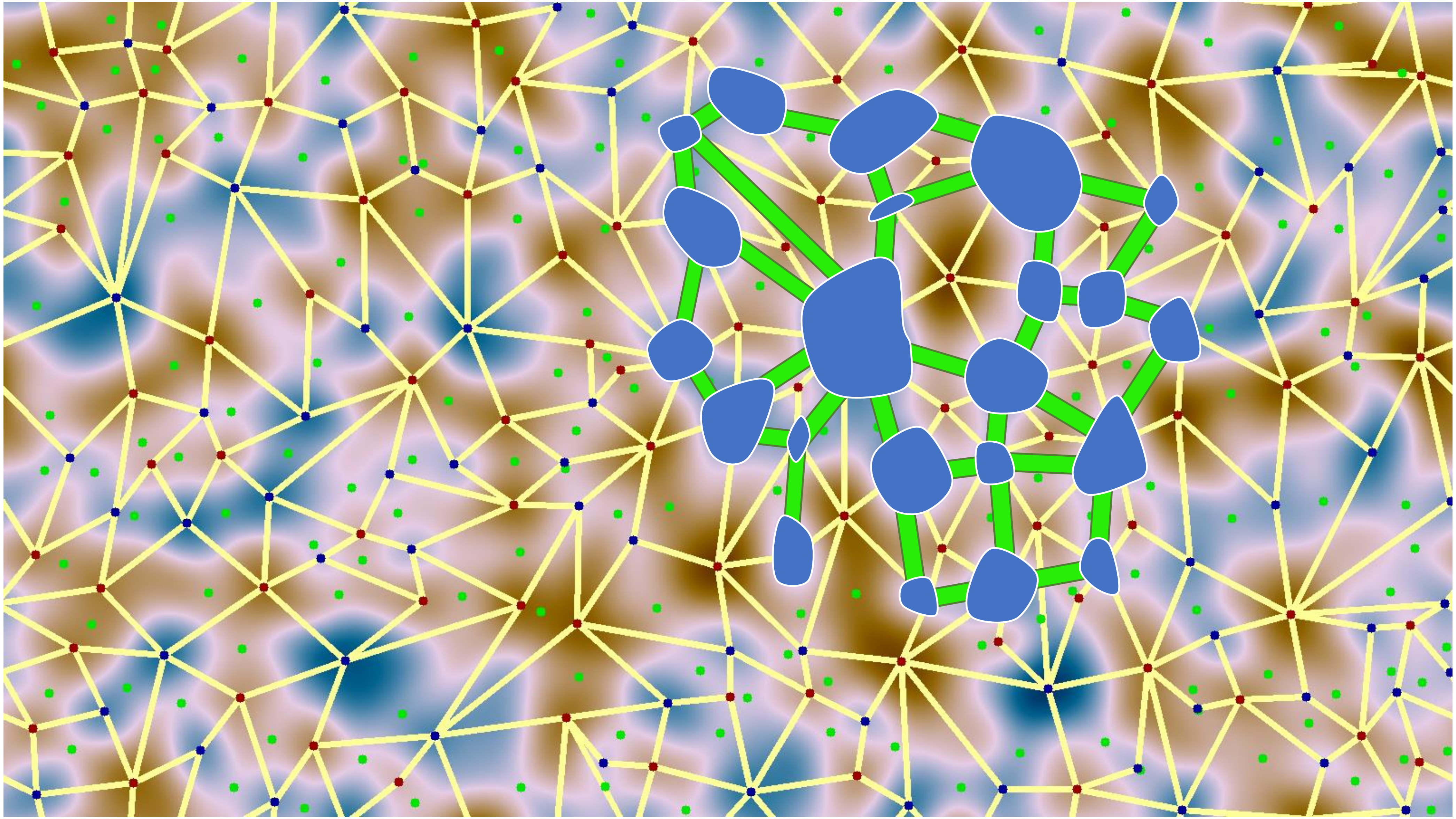
$\nu = 2.62 \pm 0.06$: H. Obuse, et.al. *PRL 109*, 206804 (2012)

$\nu = 2.56 \pm 0.01$: W. Nuding, et.al. *PRB 91*, 115107 (2015)

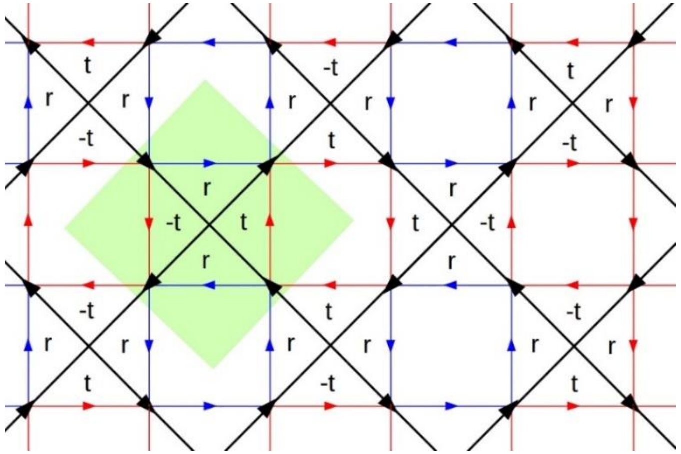
The phase randomness isn't enough

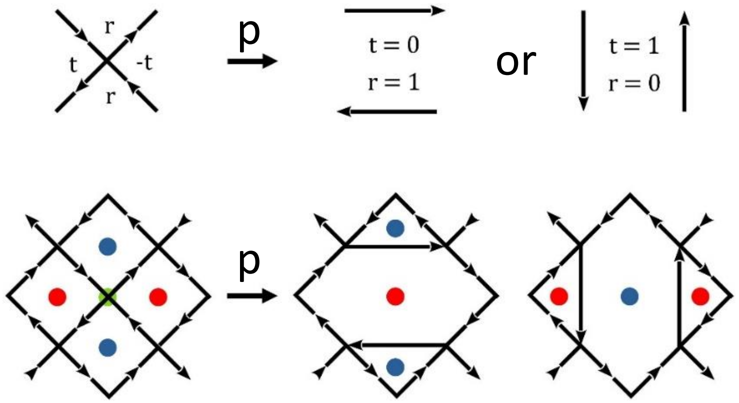
The model doesn't consider interaction

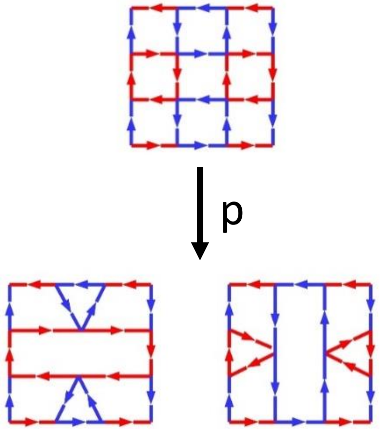
Geomerty randomness



Random network model

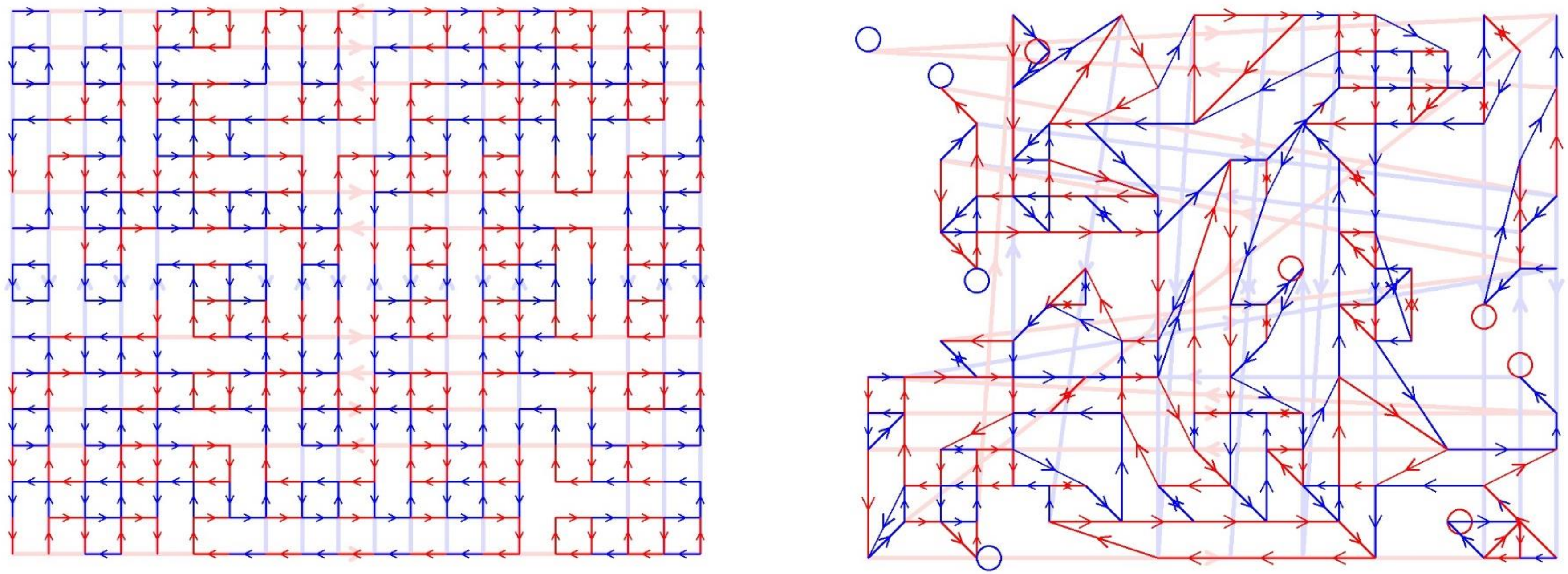




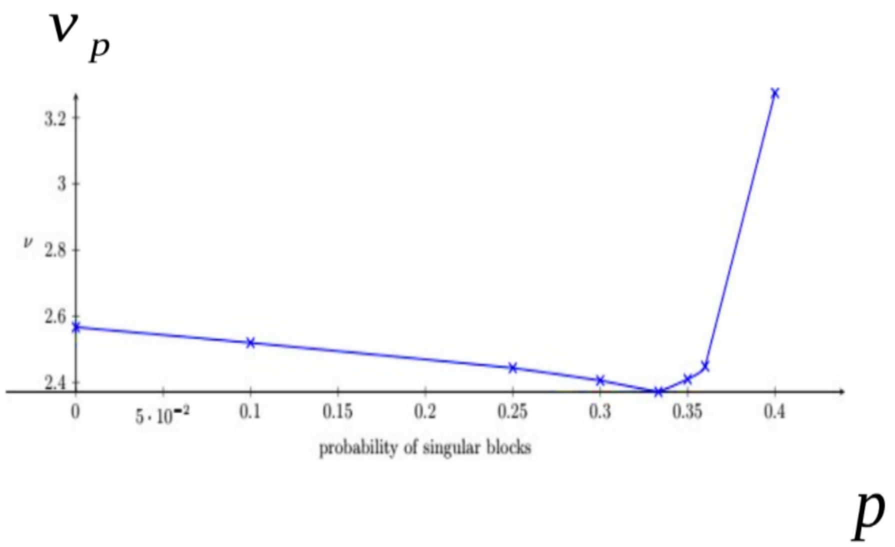


I.Gruzberg et al. *PRB 95*, 125417 (2017)

An example of a modification with $p = 0.25$

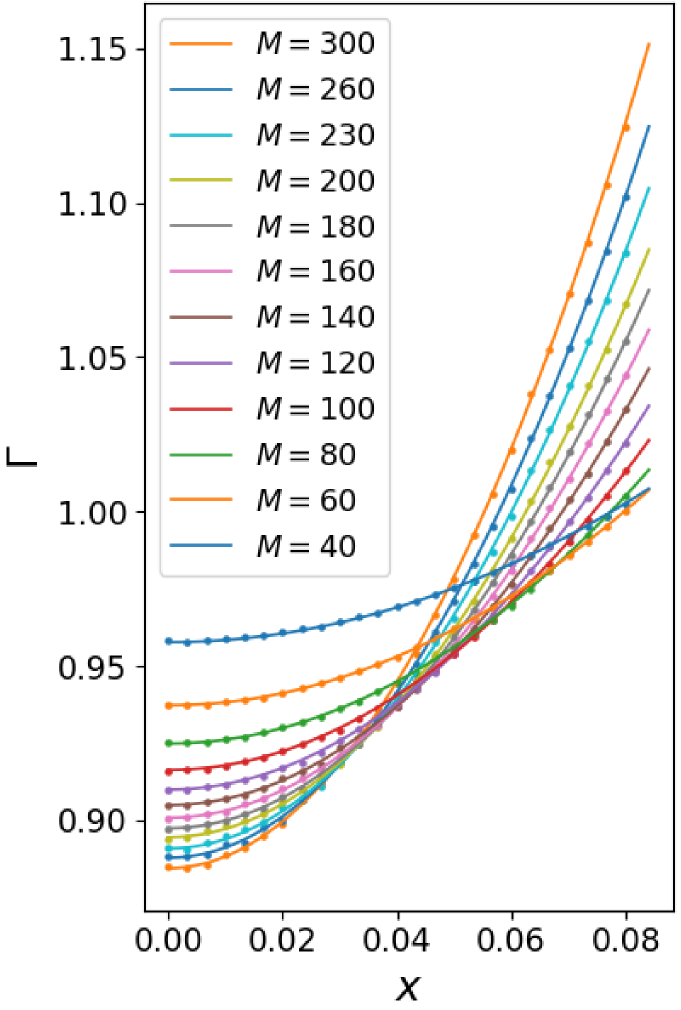


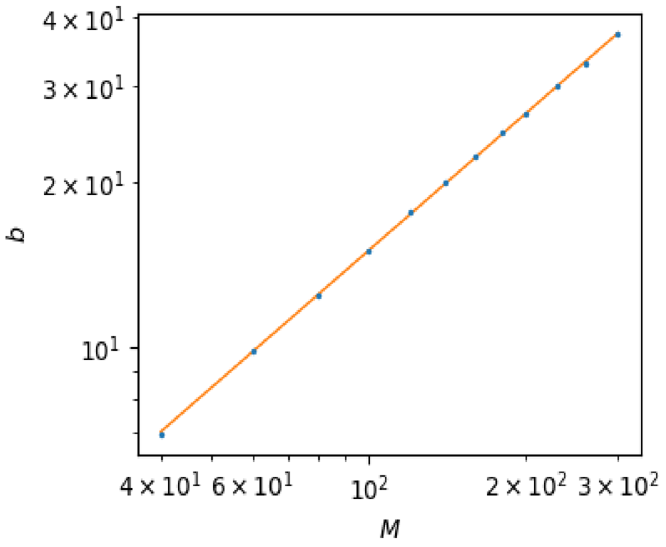
RN comparision to experiment



$$\nu(1/3) = 2.37 \pm 0.02$$

$$\nu_{exp} = 2.38 \pm 0.06$$





$$\nu(1/3) = 2.398 \pm 0.006$$

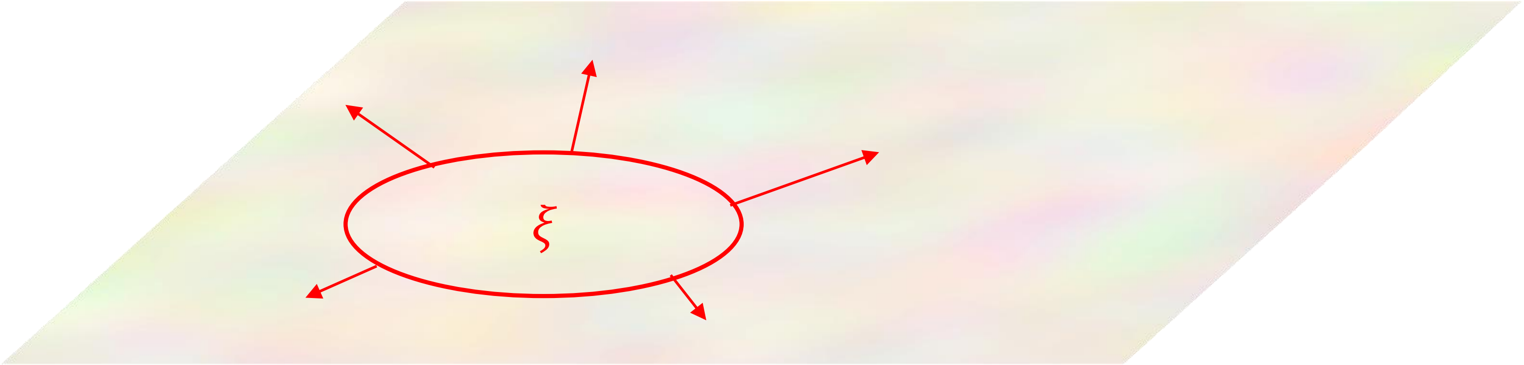
I.Gruzberg et al. *PRB 95*, 125417 (2017)

W. Nudding et.al. *PRB 100*, 140201 (2019)

H.Topchyan et al. *PRB 110*, L081112 (2024)

Harris Criterion

Can the disorder change the critical behavior?



At phase transitions correlation length is infinite

Correlated subsystems have their own critical point $x_c^{(i)}$

The larger the subsystem, the better the averaging

Approaching x_c

$\downarrow$

Increasing ξ

$\downarrow$

$x_c^{(i)}$ approaches x_c

We are faster | $x_c^{(i)}$ are faster

Subsystem phases

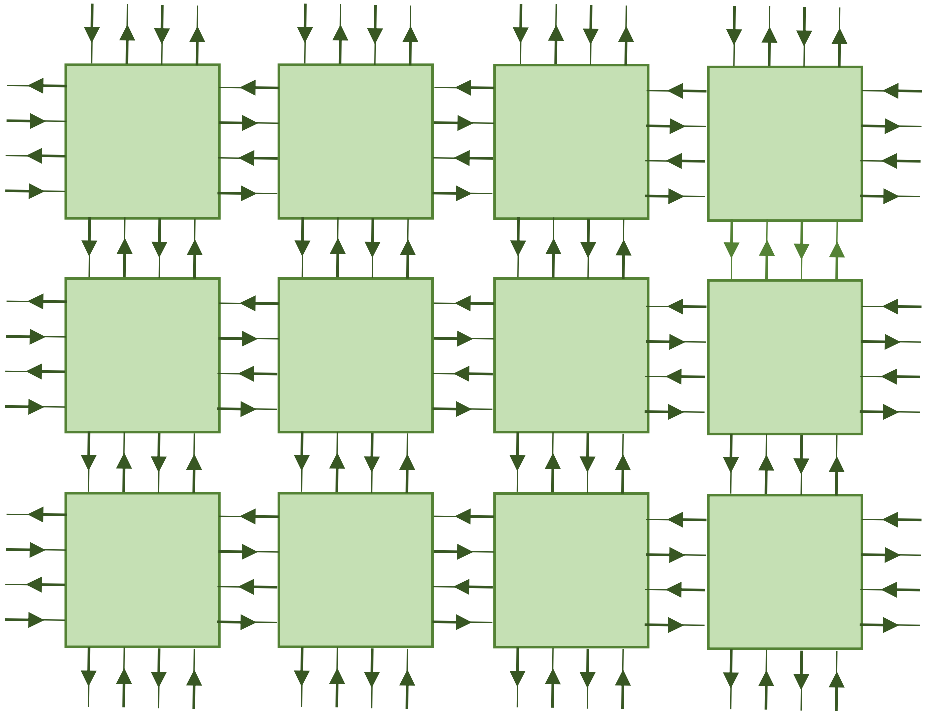
Diverge | Converge

$\downarrow$ The disorder is $\downarrow$

Significant | Irrelevant

Critical indices (such as ν) can’t change if the disorder is irrelevant

Harris Criterion



N random parameter changes:

$$x_c \to x_c \pm \Delta x_c, \qquad \Delta x_c \sim N^{-1/2}$$

Random parameters on latitce of size L in d dimensions:

$$x_c \to x_c \pm \Delta x_c, \qquad \Delta x_c \sim L^{-d/2}$$

Correlated region size:

$$\xi \sim (x - x_c)^{-\nu}$$

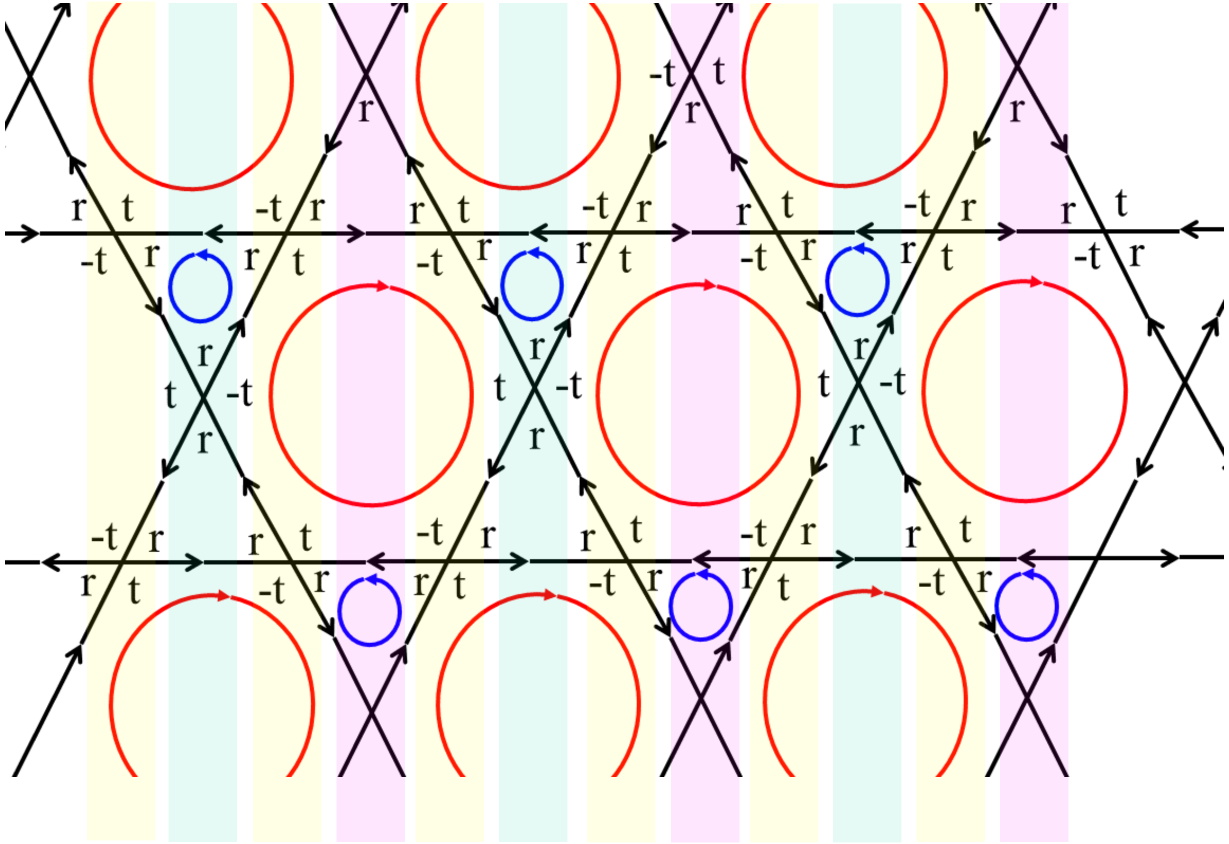
Dispersion of local critical values against our distance from the non-disordered critical point:

$$\Delta x_c \sim \xi^{-d/2} \sim (x - x_c)^{d\nu/2}$$

The critical behavior is stable against random defects if $\Delta x_c < x - x_c$ will hold when $x \to x_c$.

$$d\nu > 2$$

Hints of breaking of Harris Criterion



Kagome lattice

$$\begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}_R = \cdots \hat{U}\hat{T}_3\hat{U}\hat{T}_1\hat{U}\hat{T}_2\hat{U}\hat{T}_1 \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}_N$$

$$\nu = 2.66 \pm 0.04$$

I.Gruzberg, et.al. *PRB 102*, 121304 (2020)

A different “frozen” geomety produces same exponent value, with a significantly different fixed point.

Harris-Luck Criterion

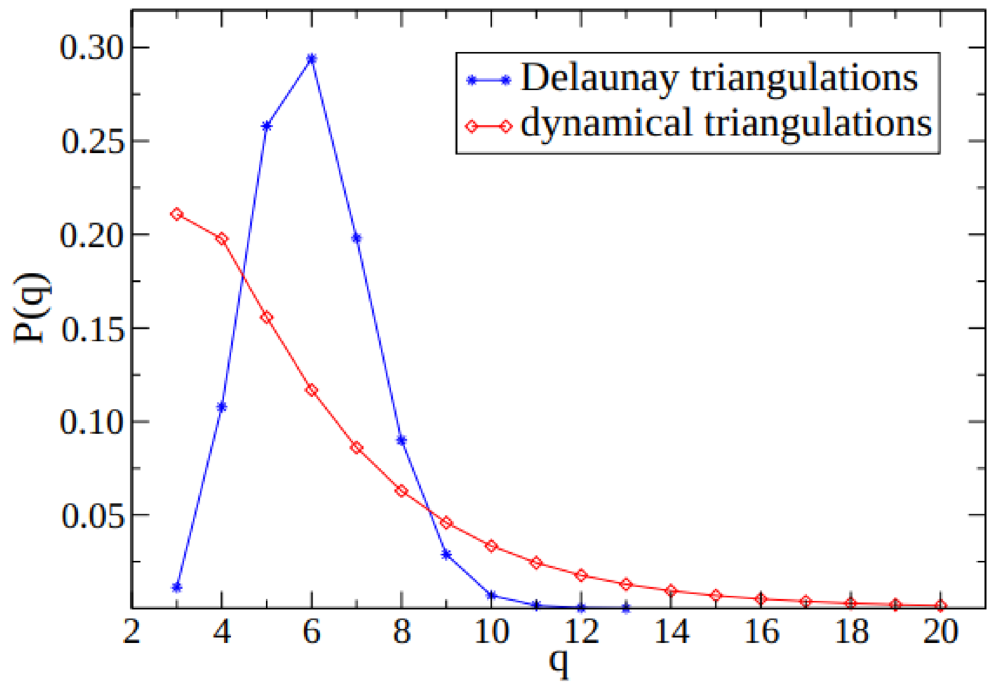
Trivial cases of Harris Criterion breaking are disorder correlations, or topological effects.

$$\Delta x_c \sim L^{-d/2} \rightarrow \Delta x_c \sim L^{-(1-\beta)d}$$

$$d\nu < 2 \rightarrow (1-\beta)d\nu < 1$$

wandering exponent

Wandering exponent is a phenome-nological descriptor of the disorder.



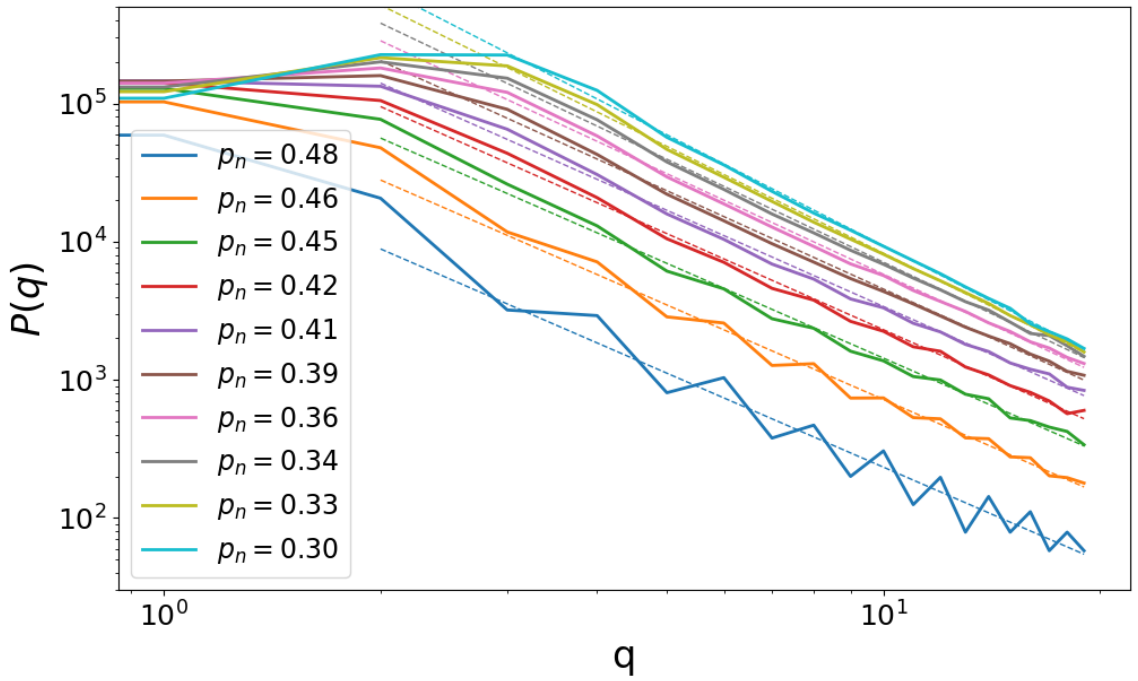
Connectivity distribution for random networks

Regular lattice	$\beta = 1/2$	$P(q) = 0, \quad q > q_0$
Random Voronoi-Delauney triangulation	$\beta = 1/2$	$P(q) \sim q^{-\sigma q}, \quad \sigma \approx 2$
Dynamical triangulation	$\beta \approx 3/4$	$P(q) \sim e^{-\sigma q}, \quad \sigma = \ln 4/3$

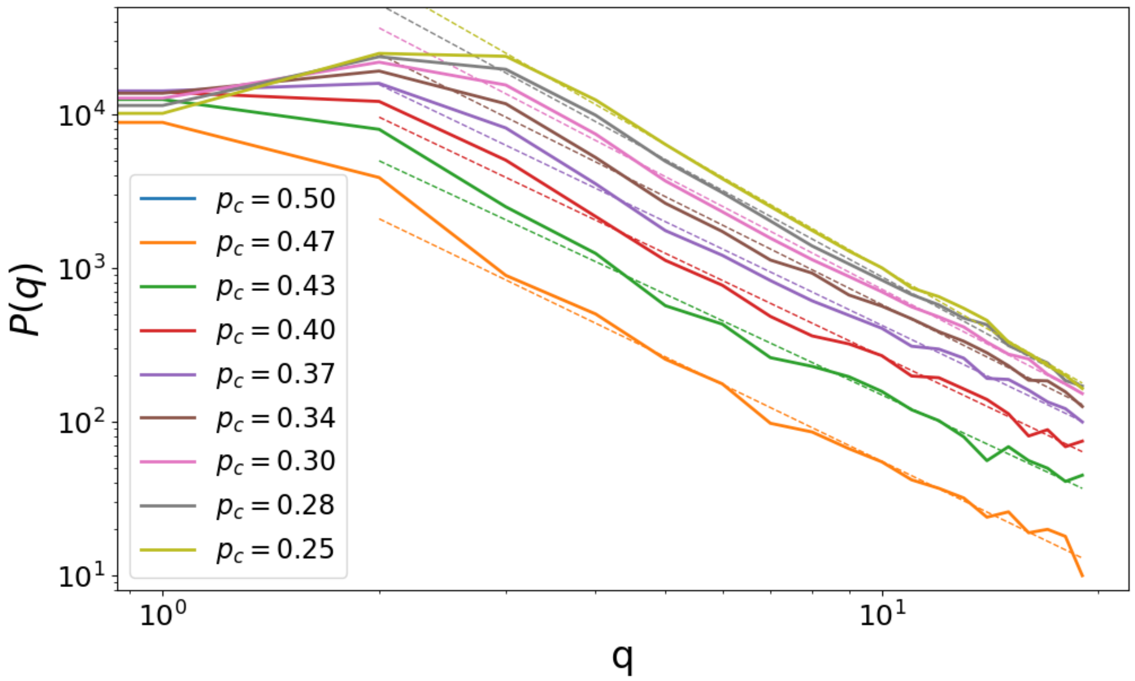
W.Janke, et.al. *PRB 69*, 144208 (2004)

Scale-free Networks

Regular lattice	$\beta = 1/2$	$P(q) = 0, \quad q > q_0$
Random Voronoi-Delauney triangulation	$\beta = 1/2$	$P(q) \sim q^{-\sigma q}, \quad \sigma \approx 2$
Dynamical triangulation	$\beta \approx 3/4$	$P(q) \sim e^{-\sigma q}, \quad \sigma = \ln 4/3$
Scale-free networks	$\beta = ?$	$P(q) \sim q^{-\sigma}$



Connectivity distribution GKNS random networks

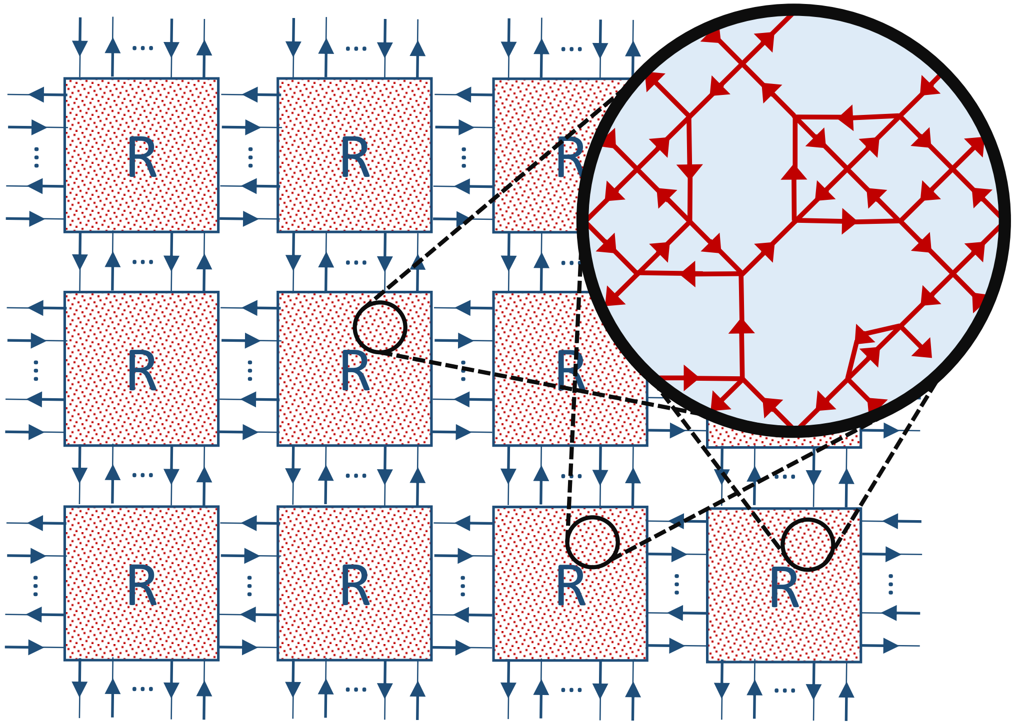


Connectivity distribution for random gaussian potentials

$\sigma \approx 2.5$

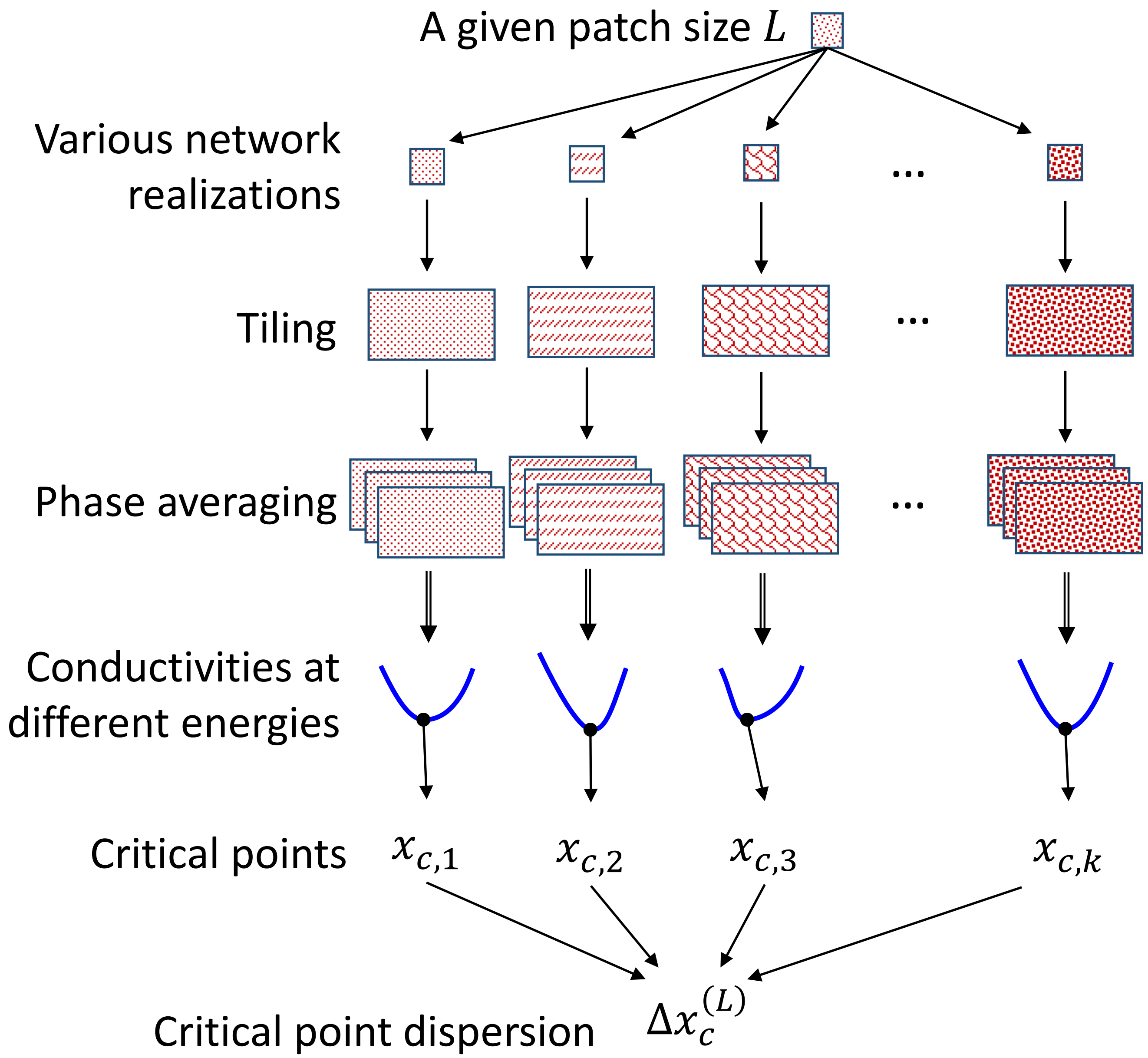
H.Topchyan, et al. *PRB 111*, L100201 (2025)

Harris Criterion for IQHE on GKNS networks

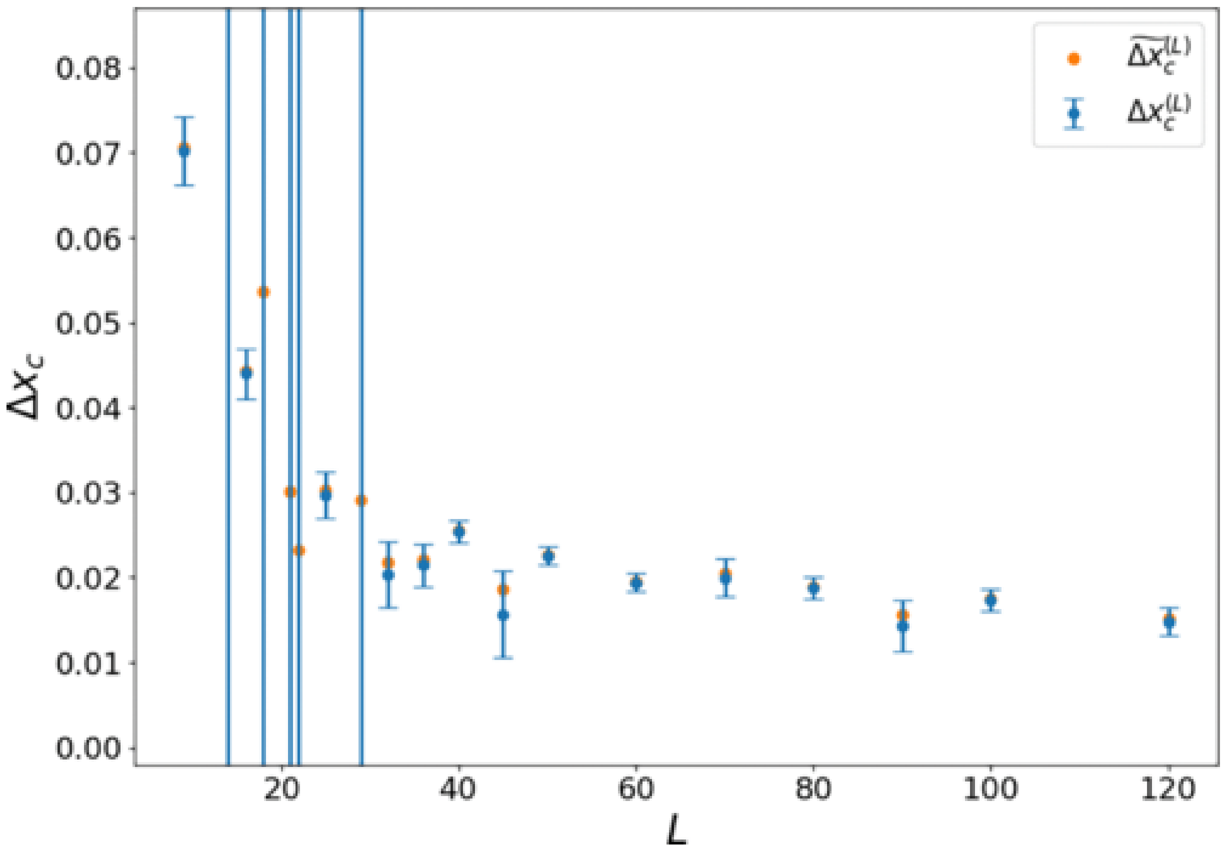


Critical value is calculated as the point of maximum conductivity

Uniformly distributed Monte-Carlo sampling is achieved through $p = 1/3$ RN generation



Harris Criterion for IQHE on KNS network



$$\Delta x_c^{(L)} = \sqrt{\widetilde{\Delta x}_c^{(L)\,2} - \boxed{2D_L^2} \pm \frac{\sqrt{2}}{2} D_L}$$

$$D_L = \sqrt{\sum_i \delta x_{c,i}^{(L)\,2}} \Big/ \#i$$

"Naïve" dispersion

Computation uncertainties

$$\widetilde{\Delta x}_c^{(L)} = \left\langle x_{c,i}^{(L)} - \left\langle x_{c,i}^{(L)} \right\rangle \right\rangle_2$$

Not considering D_L could results in studying our uncertainties, instead of the proposed model.

For larger L-s $\Delta x_c^{(L)} \sim \text{const} \sim L^0$, implying $\beta = 1$. The corresponding Harris-Luck criterion reads as $\mathbf{0 < 1}$.

The GKNS network disorder (maybe also any scale-free network?) never satisfies the irrelevance condition.

Conclusion and future work

- The mismatch between the Chalker-Coddington model and the experimantal results can be fixed through network randomzation
- GKNS disordered networks constitute a significantly different universality class of scale-free networks, which are purely studied in condensed matter context
- Additional properties such as percolation point, curvature correlations, scaling behavior, etc. can be studied understand the source of difference between GKNS networks from other random networks. This can go as far as solving the measure problem of 2 dimensional surfaces.

Thanks for attention